Chapter 5, Risk and Return

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Chapter Overview

Risk and Risk Premiums

- Rates of Return: Single Period

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} \tag{1}$$

where

$$\begin{aligned} HPR &= \text{Holding period return} \\ P_0 &= \text{Beginning price} \\ P_1 &= \text{Ending price} \\ D_1 &= \text{Dividend during period one} \end{aligned}$$

Rates of Return: Single Period Example

$$\begin{aligned} \text{Ending Price} &= \$110 \\ \text{Beginning Price} &= \$100 \\ \text{Dividend} &= \$4 \end{aligned}$$

$$HPR = \frac{\$110 - \$100 + \$4}{\$100} = 0.14, \text{ or } 14\%$$

Returns over Multiple Periods

TABLE 5.1 Quarterly cash flows and rates of return of a mutual fund

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
Assets under management at start of quarter ($ million)	1.0	1.2	2.0	0.8
Holding-period return (%)	10.0	25.0	(20.0)	20.0
Total assets before net inflows	1.1	1.5	1.6	0.96
Net inflow ($ million)*	0.1	0.5	(0.8)	0.6
Assets under management at end of quarter ($ million)	1.2	2.0	0.8	1.56

*New investment less redemptions and distributions, all assumed to occur at the end of each quarter.

Arithmetic and Geometric Averages

- Arithmetic Average

$$\bar{r} = \frac{1}{n}\sum_{s=1}^{n} r_s \tag{2}$$

- Geometric average

$$\begin{aligned} Terminal\ value =& (1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_n) \\ (1 + g)^n =& Terminal\ value \\ g =& Terminal\ value^{1/n} - 1 \end{aligned} \tag{3}$$

  - Practitioners call g time-weighted (as opposed to dollar-weighted) average return
  - The larger the swings in rates of return, the greater the discrepancy between the arithmetic and geometric averages

Example Arithmetic and Geometric Averages

From Table 5.1

- Arithmetic Average

$$\bar{r} = \frac{1}{4}\sum_{s=1}^{4} r(s) = \frac{1}{4}(0.10 + 0.25 - 0.20 + 0.20) = 0.0875 = 8.75\%$$

- Geometric average

$$(1+g)^4 = (1+0.10)\times(1+0.25)\times(1-0.20)\times(1+0.20) = 1.32$$
$$g = 1.32^{1/4} - 1 = 0.0719 = 7.19\%$$

Dollar-Weighted Return

	Quarter				
	0	1	2	3	4
Net cash flow ($ million)	−1.0	−.1	−.5	.8	−.6 + 1.56 = .96

$$0 = -1.0 + \frac{-0.1}{1 + IRR} + \frac{-0.5}{(1 + IRR)^2} + \frac{0.8}{(1 + IRR)^3} + \frac{0.96}{(1 + IRR)^4} \quad (4)$$

$$IRR = 0.0338 = 3.38\%$$

- The dollar-weighted return in this example is less than the time-weighted return of 7.19% because, the portfolio returns were higher when less money was under management.

Concept Check

CONCEPT check 5.1

A fund begins with $10 million and reports the following three-month results (with negative figures in parentheses):

	Month		
	1	2	3
Net inflows (end of month, $ million)	3	5	0
HPR (%)	2	8	(4)

Compute the arithmetic, time-weighted, and dollar-weighted average returns.

answer

Rates of Return for Different Horizons

- Horizon of T years–Example
  - Zero Coupont Bond

    Par=\$100
    Maturity $= T$
    Price=P
    Total risk free return

$$r_f(T) = \frac{100}{P(T)} - 1 \tag{5}$$

Annual Percentage Rates and Effective Annual Rates

- Annual Percentage Rate

$$APR = \text{Per-period rate} \times \text{Periods per year} = r(T) \times n$$

- EAR: Percentage increase in funds invested over a 1-year horizon

$$1 + EAR = [1 + r(T)]^n = [1 + r(T)]^{1/T} \tag{8}$$

- APR: Annualizing using simple interest

$$APR = \frac{(1 + EAR)^T - 1}{T}$$

Example 5.2 Annualized Rates of return

EXAMPLE 5.2

Annualizing Treasury-Bill Returns

Suppose you buy a $10,000 face value Treasury bill maturing in one month for $9,900. On the bill's maturity date, you collect the face value. Since there are no other interest payments, the holding-period return for this one-month investment is

$$HPR = \frac{\text{Cash income} + \text{Price change}}{\text{Initial price}} = \frac{\$100}{\$9{,}900} = .0101 = 1.01\%$$

The APR on this investment is therefore 1.01% × 12 = 12.12%. The effective annual rate is higher:

$$1 + EAR = (1.0101)^{12} = 1.1282$$

which implies that EAR = .1282 = 12.82%.

Table: EAR versus APR

Compounding Period	T	EAR = $[1 + r_f(T)]^{1/T} - 1 = .058$		APR = $r_f(T)*(1/T) = .058$	
		$r_f(T)$	APR = $[(1 + \text{EAR})^\wedge T - 1]/T$	$r_f(T)$	EAR = $(1 + \text{APR}*T)^\wedge(1/T) - 1$
1 year	1.0000	.0580	.05800	.0580	.05800
6 months	0.5000	.0286	.05718	.0290	.05884
1 quarter	0.2500	.0142	.05678	.0145	.05927
1 month	0.0833	.0047	.05651	.0048	.05957
1 week	0.0192	.0011	.05641	.0011	.05968
1 day	0.0027	.0002	.05638	.0002	.05971
Continuous			$r_{cc} = \ln(1 + \text{EAR}) = .05638$		EAR = $\exp(r_{cc}) - 1 = .05971$

Continuous compounding

- The difference between APR and EAR grows with the frequency of compounding
- As T in $[1 + T \times APR]^{1/T}$ approaches zero we approach continuous compounding

$$1 + EAR = \exp(r_{cc}) = e^{r_{cc}} \tag{9}$$
$$r_{cc} = \ln(1 + EAR)$$

Expected Return and Standard Deviation

- Expected Return

$$E(r) = \sum_{s} p(s)r(s) \tag{10}$$

where

$$\begin{aligned} p(s) &= \text{Probability of a state} \\ r(s) &= \text{Return if state occurs} \\ s &= \text{State} \end{aligned}$$

Expected Return and Standard Deviation

- Variance
  - is the expectation of the squared deviation of a random variable from its mean; measures how far a set of (random) numbers are spread out from their mean[1].

$$\sigma^2 = \sum_s p(s) \left[r(s) - E(r) \right]^2 \tag{11}$$

- Standard deviation

$$\sigma = \sqrt{\sigma^2} = \sqrt{\sum_s p(s) \left[r(s) - E(r) \right]^2}$$

[1] source: Wikipedia

Spreadsheet 5.1 Expected Returns and Standard Deviations

	A	B	C	D	E	F
1	**Spreadsheet 5.1**					
2	**Scenario analysis for the stock market**					
3	Scenario	Probability	HPR (%)	Column B x Column C	Deviation from Mean Return	Column B x Squared Deviation
4	1. Severe recession	0.05	-37	-1.85	-47.00	110.45
5	2. Mild recession	0.25	-11	-2.75	-21.00	110.25
6	3. Normal growth	0.40	14	5.60	4.00	6.40
7	4. Boom	0.30	30	9.00	20.00	120.00
8	Column sums:	Expected return =		10.00	Variance =	347.10
9		Square root of variance =			Standard deviation (%) =	18.63
10						

Web link to spreadsheet:

http://somfin.gmu.edu/courses/fnan311/Spreadsheets_5_1.xls

$E(r)$ and σ, Scenario Example 5.3

- From Spreadsheet 5.1, column HPR

$$E(r) = 0.05 \times (-0.37) + 0.25 \times (-0.11) + 0.40 \times 0.14 + 0.30 \times 0.30$$
$$E(r) = .10, \text{ or } 10.00\%$$

- The variance is computed as

$$\begin{aligned}\sigma^2 =& 0.05 \times (-0.37 - 0.10)^2 + 0.25 \times (-0.11 - 0.10)^2 \\ &+ 0.40 \times (0.14 - 0.10)^2 + 0.30 \times (0.30 - 0.10)^2 \\ =& .03471\end{aligned}$$

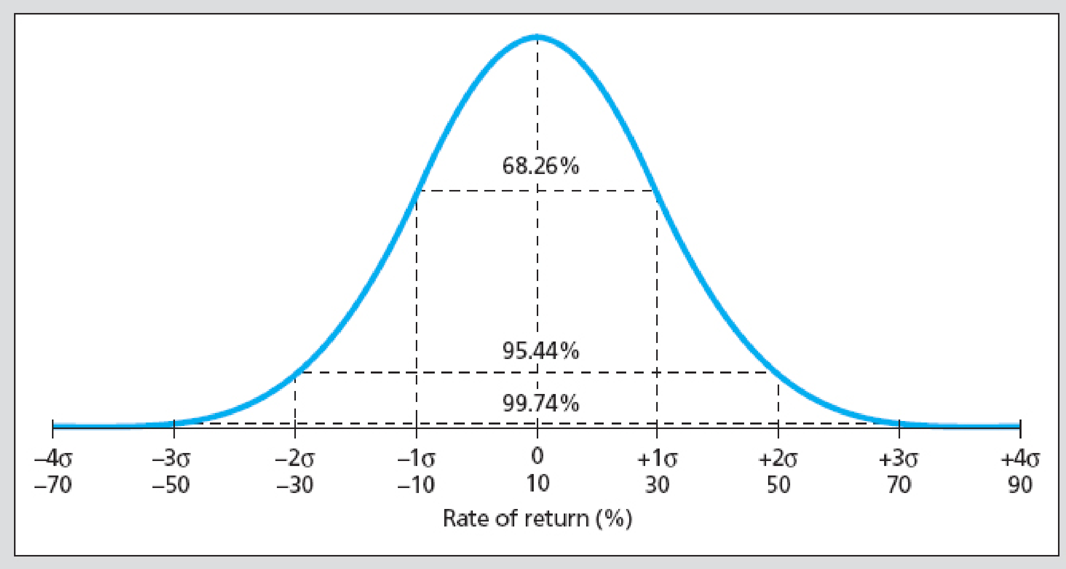
- Standard deviation is:

$$\sigma = \sqrt{\sigma^2} = \sqrt{0.03471} = 0.1863 = 18.63\%$$

The Normal Distribution

- Investment management is easier when returns are normal
  - Standard deviation is a good measure of risk when returns are symmetric
  - If security returns are symmetric, portfolio returns will be as well
  - Future scenarios can be estimated using only the mean and the standard deviation
  - The dependence of returns across securities can be summarized using only the pairwise correlation coefficients

Figure 5.1 The Normal Distribution



Measures

- Distance from the mean in units of standard deviations

$$sr = \frac{r - E(r)}{\sigma}$$

  where sr is standardized return

- recover original return using sr

$$r = E(r) + sr \times \sigma$$

  - What is a three sigma event?

Properties

1. The return on a portfolio comprising two or more assets whose returns are normally distributed also will be normally distributed.
2. The normal distribution is completely described by its mean and standard deviation. No other statistic is needed to learn about the behavior of normally distributed returns.

These two properties in turn imply this far-reaching conclusion:

3. The standard deviation is the appropriate measure of risk for a portfolio of assets with normally distributed returns. In this case, no other statistic can improve the risk assessment conveyed by the standard deviation of a portfolio.

Normality and Risk Measures

- Value at Risk (VaR)
  - Loss corresponding to a very low percentile of the entire return distribution, such as the fifth or first percentile return

$$VaR = E(r) - n_{\sigma} \times \sigma \tag{12}$$

where n_{σ} is number of standard deviations left of mean. Corresponds to a certain probability.

Concept check 5.2

a. The current value of a stock portfolio is \$23 million. A financial analyst summarizes the uncertainty about next year's holding-period return using the scenario analysis in the spreadsheet on the following slide. What are the annual holding-period returns of the portfolio in each scenario? Calculate the expected holding-period return, the standard deviation of returns, and the 5% VaR. What is the VaR of a portfolio with normally distributed returns with the same mean and standard deviation as this stock?

b. Suppose that the worst three rates of return in a sample of 36 monthly observations are 17%, 5%, and 2%. Estimate the 5% VaR.

Concept check Spreadsheet

	A	B	C	D	E
1	**Business**			**End-of-year value**	**Annual dividend**
2	**Conditions**	**Scenario, *s***	**Probability, *p***	**($ million)**	**($ million)**
3	High growth	1	0.30	35	4.40
4	Normal growth	2	0.45	27	4.00
5	No growth	3	0.20	15	4.00
6	Recession	4	0.05	8	2.00

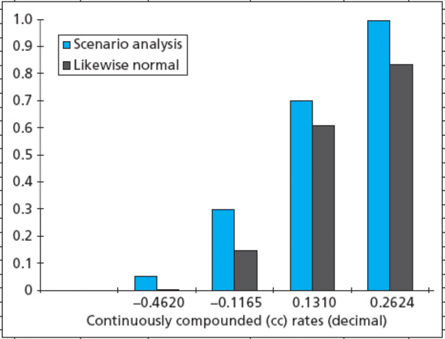
Web link to spreadsheet:

http://somfin.gmu.edu/courses/fnan311/Concept_Check_5_2.xls

Normality over Time

- Portfolios of normally distributed returns are also normally distributed
- Any sum of normally distributed variables is also normal
- Compounded returns are products, and products of normally distributed variables are not normal
- so if monthly returns are normally distributed, the annual compounded return, $(1 + r_m)^{12} - 1$, is not normally distributed.
- longer period returns will deviate substantially from normality
- Variance grows in direct proportion to time, the standard deviation grows in proportion to the square root of time.

Fig 5.2, Comparing sample distribution to normal distribution



	Scenario Analysis					Cumulative
					Squared	Corresponding
	Probability	Cumulative	Effective Rate	cc Rate (decimal)	Deviations	Normal
	0.05	0.05	–0.37	–0.4620	0.2926	0.0020
	0.25	0.3	–0.11	–0.1165	0.0382	0.1494
	0.40	0.7	0.14	0.1310	0.0027	0.6092
	0.30	1	0.30	0.2624	0.0337	0.8354
		Mean	0.10	0.0789		
				SD	0.1881	
	VaR Scenario Analysis		–0.37	–0.4620		
	VaR corresponding Normal		–0.2058	–0.2304		

Deviation from Normality and Value at Risk

- Fig. 5.2 compares known points from the cumulative distribution of the scenario analysis next to the corresponding points from a likewise normal distribution
- The VaR from the scenario analysis, 37%, is far worse than the VaR derived from the corresponding normal distribution, 20.58%.
  - the scenario analysis entails a higher probability of extreme losses than would be consistent with a normal distribution
- On the other hand, the normal distribution allows for the possibility of extremely large returns, beyond the maximum return of 30% envisioned in the scenario analysis.
- We conclude that the scenario analysis has a distribution that is skewed to the left compared to the normal.

Skewness and Kurtosis

- When returns deviate from normality we need to consider skewness and kurtosis

$$\text{Skewness} = Average\left[\frac{(R - \bar{R})^3}{\hat{\sigma}^3}\right] \tag{13}$$

$$\text{Kurtosis} = Average\left[\frac{(R - \bar{R})^4}{\hat{\sigma}^4}\right] \tag{14}$$

Time Series Analysis of Past Returns

- True means and variances are unobservable because we don't actually know possible scenarios like the one in the examples
- So we must estimate them (the means and variances, not the scenarios)

Estimating Variance and Standard Deviation

- Estimated Variance
  - Expected value of squared deviations

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{s=1}^{n} [r(s) - \bar{r}]^2 \tag{16}$$

- Unbiased estimated standard deviation

$$\hat{\sigma}^2 = \frac{n}{n-1} \times \frac{1}{n} \sum_{s=1}^{n} [r(s) - \bar{r}]^2$$

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{s=1}^{n} [r(s) - \bar{r}]^2} \tag{17}$$

Normality and Risk Measures

- What if excess returns are not normally distributed?
  - Standard deviation is no longer a complete measure of risk
  - Sharpe ratio is not a complete measure of portfolio performance
  - Need to consider skewness and kurtosis

Figure: Normal and Skewed Distributions

Mean =6%

SD =17%

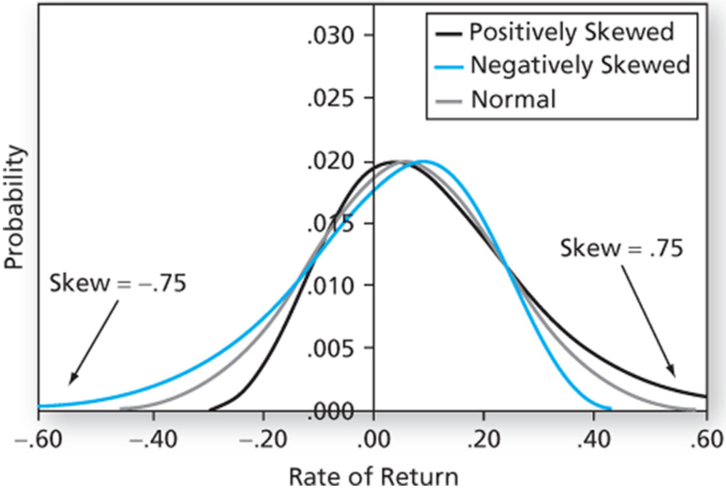
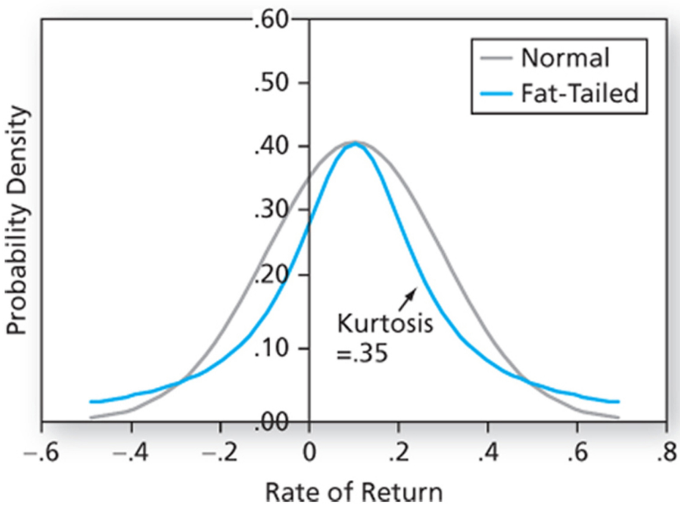


Figure: Normal and Fat-Tailed Distributions

Mean =10%

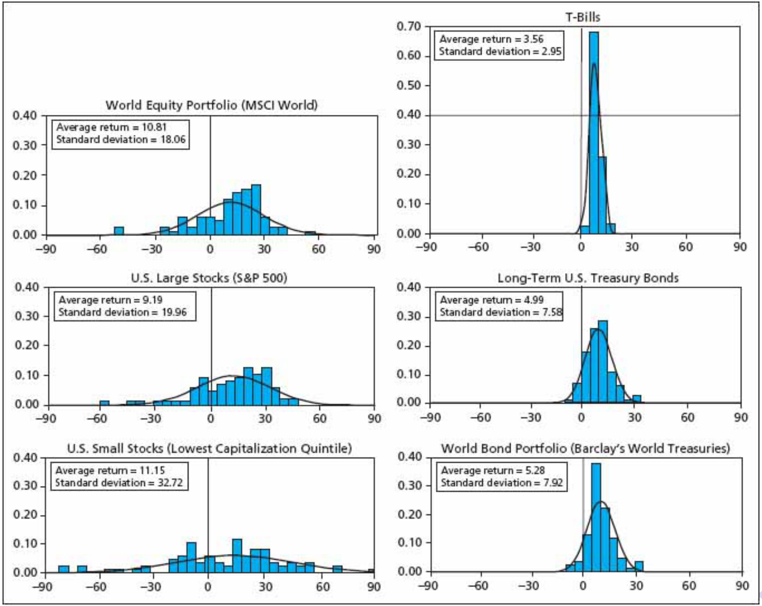
SD =20%



Historic Returns on Risk Portfolios

- The second half of the 20th century, politically and economically the most stable sub-period, offered the highest average returns
- Firm capitalization is highly skewed to the right: Many small but a few gigantic firms
- Average realized returns have generally been higher for stocks of small rather than large capitalization firms

Figure 5.3



World Equity Portfolio (MSCI World)
Average return = 10.81
Standard deviation = 18.06
T-Bills
Average return = 3.56
Standard deviation = 2.95
U.S. Large Stocks (S&P 500)
Average return = 9.19
Standard deviation = 19.96
Long-Term U.S. Treasury Bonds
Average return = 4.99
Standard deviation = 7.58
U.S. Small Stocks (Lowest Capitalization Quintile)
Average return = 11.15
Standard deviation = 32.72
World Bond Portfolio (Barclay's World Treasuries)
Average return = 5.28
Standard deviation = 7.92
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70
–90 –60 –30 0 30 60 90

Historic Returns on Risk Portfolios

- Normal distribution is generally a good approximation of portfolio returns
  - VaR indicates no greater tail risk than is characteristic of the equivalent normal
  - The ES does not exceed 0.41 of the monthly SD, presenting no evidence against the normality
- However
  - Negative skew is present in some of the portfolios some of the time, and positive kurtosis is present in all portfolios all the time

Table 5.2 Historic Returns

TABLE 5.2 Annual rate-of-return statistics for diversified portfolios for 1926–2010 and three subperiods (%)

	World Portfolio		U.S. Market		
	Equity Return in U.S. Dollars	Bond Return in U.S. Dollars	Small Stocks	Large Stocks	Long-Term T-Bonds
Total Return—Geometric Average					
1926–2010	9.21	5.42	11.80	9.62	5.12
1926–1955	8.31	2.54	11.32	9.66	3.46
1956–1985	10.28	5.94	13.81	9.52	4.64
1986–2010	9.00	8.34	9.99	9.71	7.74
Total Real Return—Geometric Average					
1926–2010	6.03	2.35	8.54	6.43	2.06
1926–1955	6.86	1.16	9.82	8.18	2.07
1956–1985	5.23	1.09	8.60	4.51	−0.15
1986–2010	5.99	5.36	6.96	6.68	4.77

Table 5.2 Historic Excess Returns

Excess Return Statistics					
Arithmetic average					
1926–2010	7.22	2.09	13.91	8.00	1.76
1926–1955	9.30	1.75	20.02	11.67	2.43
1956–1985	5.55	0.38	12.18	5.01	−0.87
1986–2010	6.74	4.54	8.66	7.19	4.11
Standard deviation					
1926–2010	18.98	8.50	37.56	20.70	7.93
1926–1955	21.50	8.10	49.25	25.40	4.12
1956–1985	16.33	8.42	32.31	17.58	8.29
1986–2010	19.27	8.81	25.82	17.83	10.07
Minimum (lowest excess return)					
1926–2010	−41.97	−18.50	−55.34	−46.65	−13.43
1926–1955	−41.03	−13.86	−55.34	−46.65	−6.40
1956–1985	−32.49	−18.50	−45.26	−34.41	−13.09
1986–2010	−41.97	−11.15	−41.47	−38.44	−13.43
Maximum (highest excess return)					
1926–2010	70.51	28.96	152.88	54.26	26.07
1926–1955	70.51	28.96	152.88	54.26	10.94
1956–1985	35.25	26.40	99.94	42.25	24.96
1986–2010	36.64	24.40	73.73	32.11	26.07

Table 5.2 Deviations from Normal Distribution

	World Portfolio		U.S. Market		
	Equity Return in U.S. Dollars	Bond Return in U.S. Dollars	Small Stocks	Large Stocks	Long-Term T-Bonds
Deviation from the Normal Distribution*					
Kurtosis					
1926–2010	1.49	1.01	0.65	1.05	0.24
1926–1955	1.88	3.05	0.03	0.97	−0.24
1956–1985	0.25	1.52	−0.08	0.04	0.99
1986–2010	1.85	−0.31	0.53	1.93	−0.45
Skew					
1926–2010	−0.83	0.44	−0.40	−0.86	0.16
1926–1955	−0.67	0.64	−0.49	−1.01	−0.20
1956–1985	−0.61	0.44	−0.31	−0.52	0.79
1986–2010	−1.36	0.26	−0.45	−1.30	−0.26

Table 5.2 Performance

	World Portfolio		U.S. Market		
	Equity Return in U.S. Dollars	Bond Return in U.S. Dollars	Small Stocks	Large Stocks	Long-Term T-Bonds
Deviation from the Normal Distribution*					
Kurtosis					
1926–2010	1.49	1.01	0.65	1.05	0.24
1926–1955	1.88	3.05	0.03	0.97	−0.24
1956–1985	0.25	1.52	−0.08	0.04	0.99
1986–2010	1.85	−0.31	0.53	1.93	−0.45
Skew					
1926–2010	−0.83	0.44	−0.40	−0.86	0.16
1926–1955	−0.67	0.64	−0.49	−1.01	−0.20
1956–1985	−0.61	0.44	−0.31	−0.52	0.79
1986–2010	−1.36	0.26	−0.45	−1.30	−0.26

The Reward-to-Volatility (Sharpe) Ratio

- Excess Return
  - The difference in any particular period between the actual rate of return on a risky asset and the actual risk-free rate
- Risk Premium
  - The difference between the expected HPR on a risky asset and the risk-free rate
- Sharpe Ratio

$$\text{Sharpe Ratio} = \frac{\text{Risk premium}}{\text{SD of excess return}} = \frac{E(r_P) - r_f}{\sigma_P} \tag{18}$$

Interest Rate Determinants

- Supply
  - Households
- Demand
  - Businesses
- Government's net demand
  - Federal Reserve actions

Real and Nominal Interest Rates

- Nominal interest rate (rn)
  - Growth rate of your money
- Real interest rate (rr)
  - Growth rate of your purchasing power
- Government's net demand
  - Federal Reserve actions

Approximate relationship

$$rr \approx rn - i \tag{19}$$

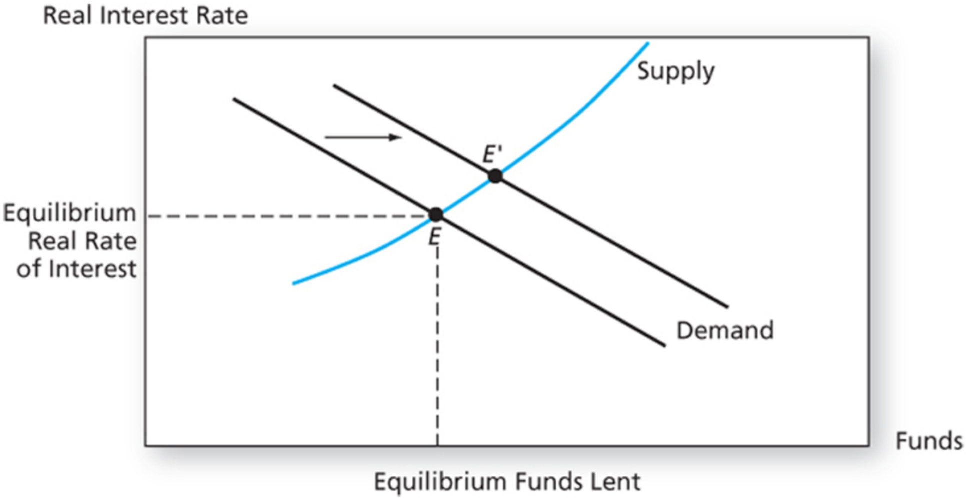
Exact relationship

$$(1 + rn) = (1 + rr)(1 + i) \tag{20}$$

$$rr = \frac{rn - i}{1 + i} \tag{21}$$

where i is the inflation rate

Figure 5.1 Equilibrium Interest Rate



Equilibrium Nominal Rate of Interest

- As the inflation rate increases, investors will demand higher nominal rates of return
- If $E(i)$ denotes current expectations of inflation, then we get the Fisher Equation:

$$rn = rr + E(i) \tag{22}$$

Taxes and the Real Rate of Interest

- Tax liabilities are based on nominal income
  - Given a tax rate, t, and nominal interest rate, rn, the real after-tax rate is:

$$rr_{AT} \approx rn(1-t) - i = (rr+i)(1-t) - i = rr(1-t) - it \qquad (23)$$

- The after-tax real rate of return falls as the inflation rate rises

Table T-Bills, Inflation Rates, and St. deviations

	Annualized Average Rates			Annualized Standard Deviations*		
	T-Bills	Inflation	Real T-Bill	T-Bills	Inflation	Real T-Bill
All months	3.55	3.04	0.52	2.95	4.06	3.95
First half	1.79	1.74	0.10	1.56	4.66	4.98
Recent half	5.35	4.36	0.95	3.02	2.82	2.44

*Annualized standard deviations include the effect of serial correlations at lags 1–11 months.
Sources: *Annual rates of return from rolling over 1-month T-bills:* Kenneth French; *annual inflation rates:* Bureau of Labor Statistics.

Bills and Inflation

- Moderate inflation can offset most of the nominal gains on low-risk investments
- A dollar invested in T-bills from 1926-2012 grew to $20.25 but with a real value of only $1.55
- Negative correlation between real rate and inflation rate means the nominal rate doesn't fully compensate investors for increased in inflation

Figure T-Bills and Inflation

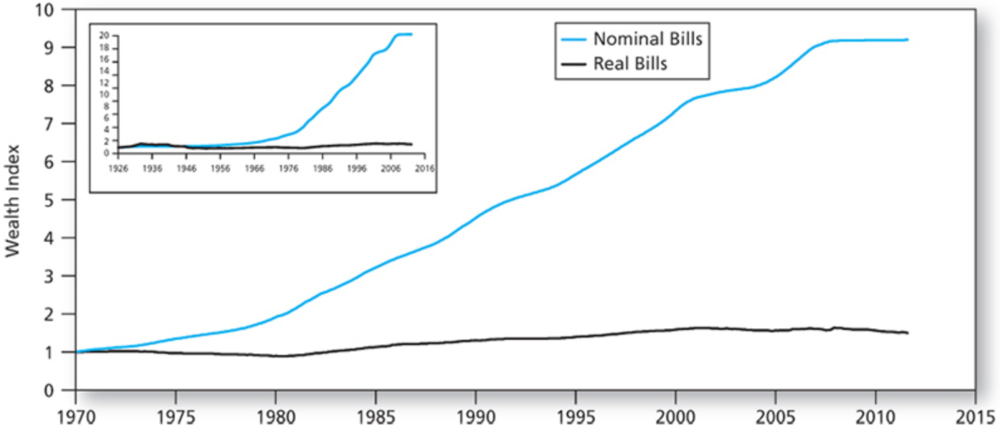


Figure: Interest Rates and Inflation

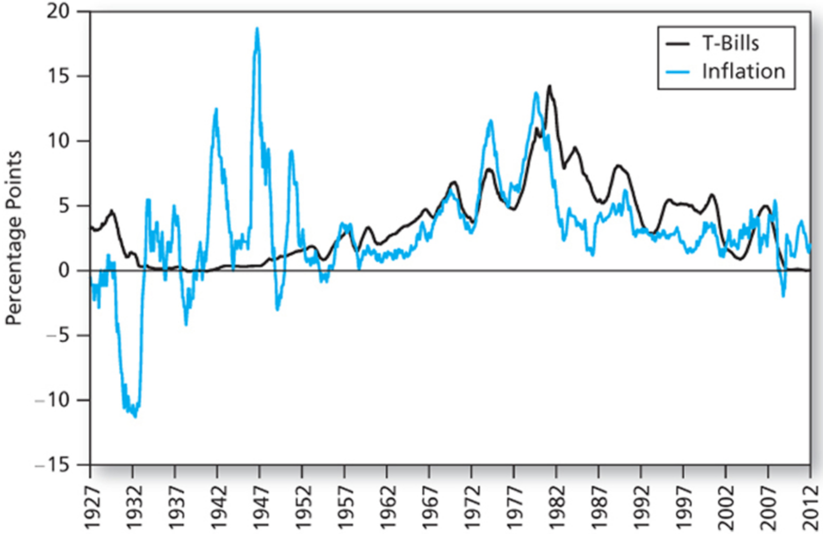


Figure: Returns Around the World

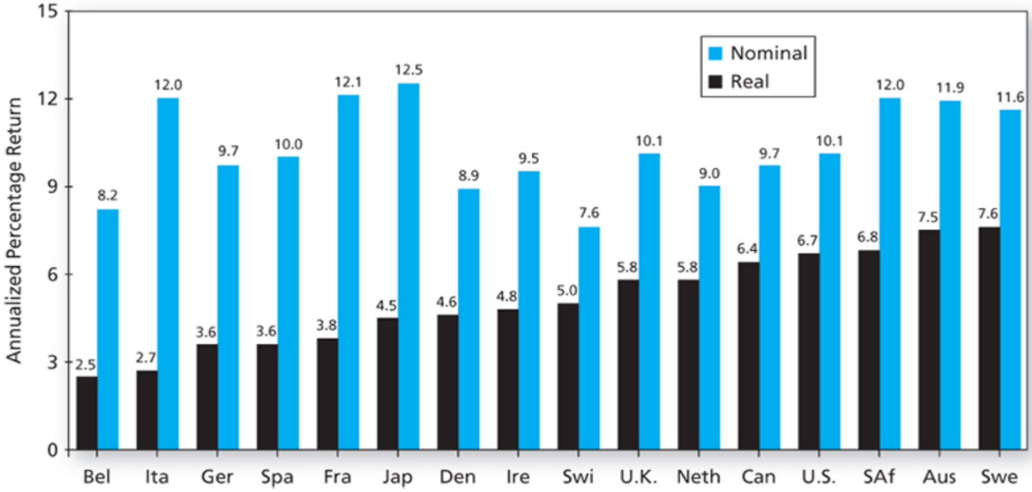
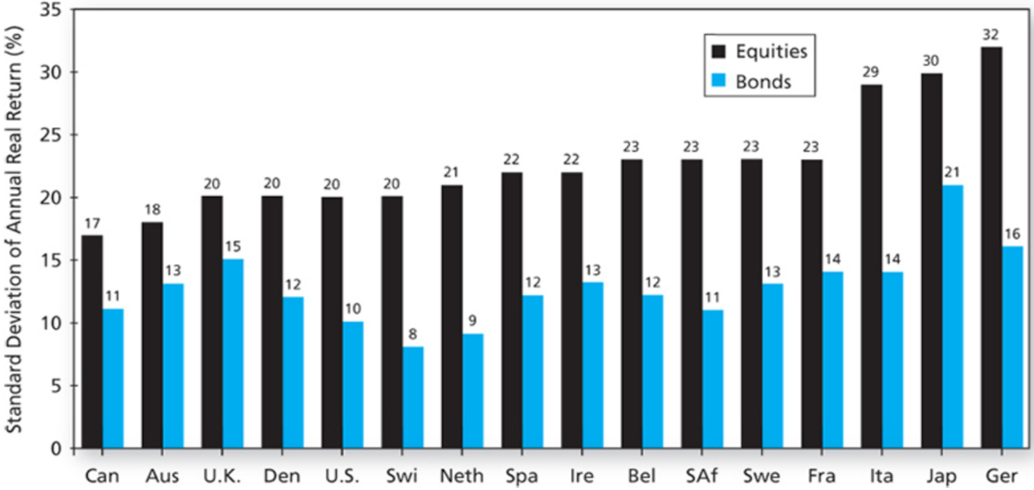


Figure SD of Real Equity and Bond Portfolios



Portfolios of One Risky Asset and a Risk-Free Asset

- It's possible to create a complete portfolio by splitting investment funds between safe and risky assets

$$\text{Let} \qquad y = \text{Portion allocated to the risky portfolio, } P$$
$$(1-y) = \text{Portion to be invested in risk-free asset, } F$$

$$r_f = 7\% \qquad \sigma_{rf} = 0\%$$
$$E(r_P) = 15\% \qquad \sigma_P = 22\%$$

- The expected return on the complete portfolio

$$E(r_C) = yE(r_P) + (1-y)r_f = r_f + y[E(r_P) - r_f] = 7 + y(15-7) \tag{3}$$

- The risk of the complete portfolio is

$$\sigma_C = y\sigma_P = 22y \tag{4}$$

One Risky Asset and a Risk-Free Asset: Example

- Rearrange and substitute $y = \sigma_C / \sigma_P$

$$E(r_C) = r_f + y[E(r_P - r_f]$$
$$= r_f + \frac{\sigma_C}{\sigma_P}[E(r_P - r_f] = 7 + \frac{8}{22}\sigma_C \qquad (5)$$

- and the slope is:

$$S = \frac{E(r_P) - r_f}{\sigma_P} = \frac{8}{22} \qquad (6)$$

Concept Check

CONCEPT check 5.6

What are the expected return, risk premium, standard deviation, and ratio of risk premium to standard deviation for a complete portfolio with $y = .75$?

answer

Example 5.7

EXAMPLE 5.7

Levered Complete Portfolios

Suppose the investment budget is \$300,000, and an investor borrows an additional \$120,000, investing the \$420,000 in the risky asset. This is a levered position in the risky asset, which is financed in part by borrowing. In that case

$$y = \frac{420{,}000}{300{,}000} = 1.4$$

and $1 - y = 1 - 1.4 = -.4$, reflecting a short position in the risk-free asset, or a borrowing position. Rather than lending at a 7% interest rate, the investor borrows at 7%. The portfolio rate of return is

Example 5.7

$$E(r_C) = 7 + (1.4 \times 8) = 18.2$$

Another way to find this portfolio rate of return is as follows: You expect to earn $63,000 (15% of $420,000) and pay $8,400 (7% of $120,000) in interest on the loan. Simple subtraction yields an expected profit of $54,600, which is 18.2% of your investment budget of $300,000. Therefore, $E(r_C) = 18.2\%$.

Your portfolio still exhibits the same reward-to-volatility ratio:

$$\sigma_C = 1.4 \times 22 = 30.8$$

$$S = \frac{E(r_C) - r_f}{\sigma_C} = \frac{11.2}{30.8} = .36$$

As you might have expected, the levered portfolio has both a higher expected return and a higher standard deviation than an unlevered position in the risky asset.

Risk Tolerance and Asset Allocation

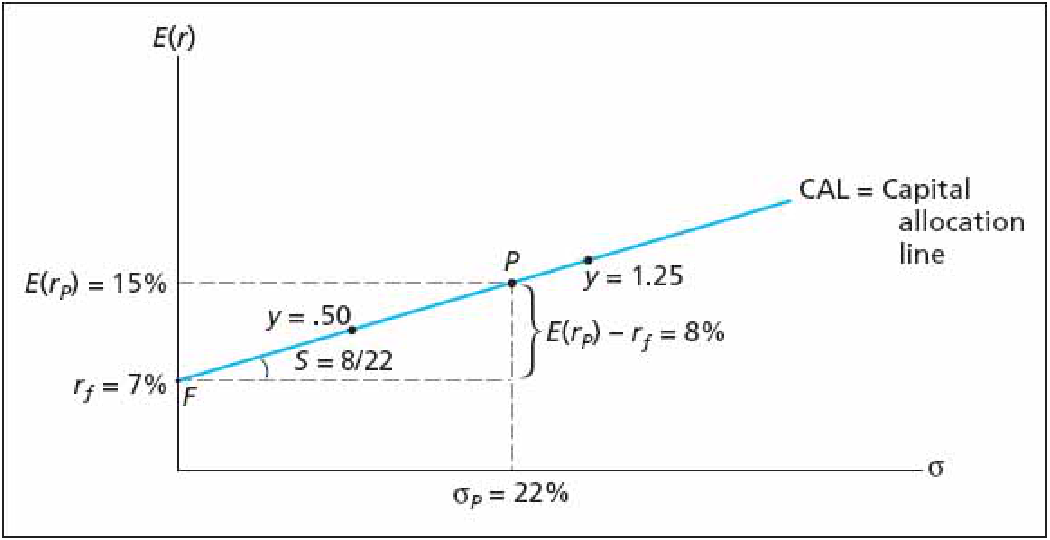
- The investor must choose one optimal portfolio, C, from the set of feasible choices
- Expected return of the complete portfolio:

$$E(r_C) = r_f + y[E(r_P - r_f]$$

- Variance:

$$\sigma_C^2 = y^2 \sigma_P^2$$

Figure 5.6 The Investment Opportunity Set



Finding the Optimal Complete Portfolio

- The optimal position in the risky asset is:

$$y^* = \frac{E(r_P) - r_f}{A\sigma_p^2} \tag{7}$$

Example Capital Allocation

- Using our numerical example [$rf = 7\%, E(r_P) = 15\%, \sigma_P = 22\%$], and expressing all returns as decimals, the optimal solution for an investor with a coefficient of risk aversion $A = 4$ is:

$$y^* = \frac{0.15 - 0.07}{4 \times 0.22^2} = 0.41$$

- With 41% invested in the risky portfolio, the expected return and standard deviation of the complete portfolio are

$$E(r_C) = 7 + [0.41 \times (15 - 7)] = 10.28\%$$
$$\sigma_C = 0.41 \times 22 = 9.02\%$$

- The risk premium of the complete portfolio is $E(r_C) r_f = 3.28\%$, which is obtained by taking on a portfolio with a standard deviation of 9.02%.
- Notice that $3.28/9.02 = 0.36$, which is the reward-to-volatility (Sharpe) ratio

Passive Strategies and the Capital Market Line

- Securities are fairly priced
- Select a diversified portfolio of common stocks
- Indexing–chose an index mutual fund or an ETF.
- The capital market line (CML)
- Historically the CML has a Sharpe ration of 0.4. Can your active strategy beat that?

Table 5.4

TABLE 5.4 Excess return statistics for the S&P 500

	Excess Return (%)			
	Average	**Std Dev**	**Sharpe Ratio**	**5% VaR**
1926–2010	8.00	20.70	.39	−36.86
1926–1955	11.67	25.40	.46	−53.43
1956–1985	5.01	17.58	.28	−30.51
1986–2010	7.19	17.83	.40	−42.28