Chapter 7, The Capital Asset Pricing Model

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Chapter Overview

The Capital Asset Pricing Model (CAPM)

- It is the equilibrium model that underlies all modern financial theory
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development

Assumptions

- Investors optimize portfolios a la Markowitz
- Investors use identical input list for efficient frontier
- Same risk-free rate, tangent CAL and risky portfolio
- Market portfolio is aggregation of all risky portfolios and has same weights
- Investors are price takers
- No transaction costs
- No taxes

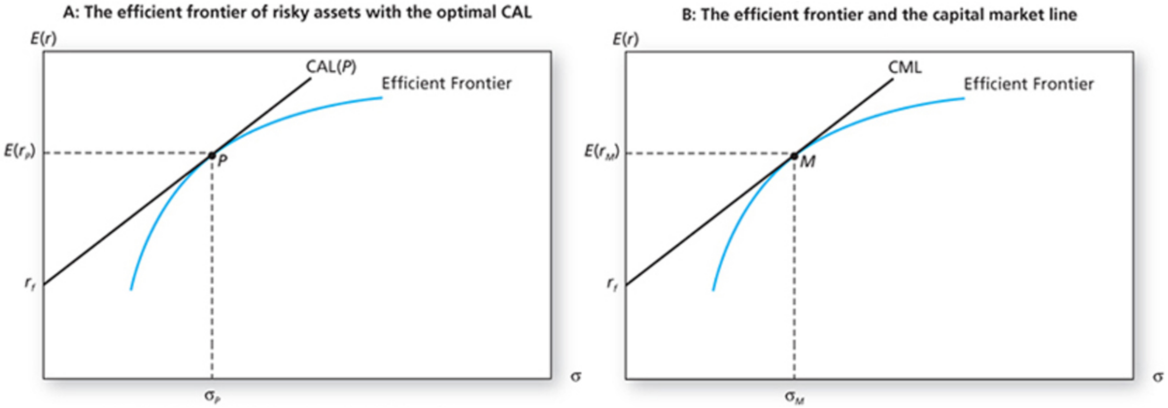
Assumptions of the CAPM

1. Individual behavior
   a. Investors are rational, mean-variance optimizers.
   b. Their planning horizon is a single period.
   c. Investors have homogeneous expectations (identical input lists).
2. Market structure
   a. All assets are publicly held and trade on public exchanges, short positions are allowed, and investors can borrow or lend at a common risk-free rate.
   b. All information is publicly available.
   c. No taxes.
   d. No transaction costs.

Resulting Equilibrium Conditions

- All investors will hold the same portfolio for risky assets – market portfolio
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value

Figure The Efficient Frontier and the CML



Market Risk Premium

- The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} \qquad (1)$$

$$E(R_M) = \bar{A}\sigma_M^2 \qquad (2)$$

where

$E(R_M) = E(r_M) - r_f$ is the risk premium (expected excess return) on the market

σ_M^2 is the variance of the market portfolio

$\bar{A}$ is the average degree of risk aversion across investors

Example

EXAMPLE 7.1

Market Risk, the Risk Premium, and Risk Aversion

Suppose the risk-free rate is 5%, the average investor has a risk-aversion coefficient of $\bar{A} = 2$, and the standard deviation of the market portfolio is 20%. Then, from Equation 7.1, we estimate the equilibrium value of the market risk premium[1] as $2 \times .20^2 = .08$. So the expected rate of return on the market must be

$$E(r_M) = r_f + \text{Equilibrium risk premium}$$
$$= .05 + .08 = .13 = 13\%$$

If investors were more risk averse, it would take a higher risk premium to induce them to hold shares. For example, if the average degree of risk aversion were 3, the market risk premium would be $3 \times .20^2 = .12$, or 12%, and the expected return would be 17%.

Concept check

CONCEPT check 7.2

Historical data for the S&P 500 Index show an average excess return over Treasury bills of about 7.5% with standard deviation of about 20%. To the extent that these averages approximate investor expectations for the sample period, what must have been the coefficient of risk aversion of the average investor? If the coefficient of risk aversion were 3.5, what risk premium would have been consistent with the market's historical standard deviation?

Return and Risk for Individual Securities

- The risk premium on individual securities is a function of the individual security's contribution to the risk of the market portfolio.
- An individual security's risk premium is a function of the covariance of returns with the assets that make up the market portfolio.

GE Example

- Contribution of GE to market variance

Portfolio Weights	w_1	w_2	...	w_{GE}	...	w_n
w_1	$Cov(R_1, R_1)$	$Cov(R_1, R_2)$	...	$Cov(R_1, R_{GE})$	...	$Cov(R_1, R_n)$
w_2	$Cov(R_2, R_1)$	$Cov(R_2, R_2)$	...	$Cov(R_2, R_{GE})$	...	$Cov(R_2, R_n)$
⋮	⋮	⋮		⋮		⋮
w_{GE}	$Cov(R_{GE}, R_1)$	$Cov(R_{GE}, R_2)$	...	$Cov(R_{GE}, R_{GE})$	...	$Cov(R_{GE}, R_n)$
⋮	⋮	⋮		⋮		⋮
w_n	$Cov(R_n, R_1)$	$Cov(R_n, R_2)$	...	$Cov(R_n, R_{GE})$	...	$Cov(R_n, R_n)$

GE Example

- Covariance of GE return with the market portfolio:

$$\sum_{i=1}^{n} w_i Cov(R_i, R_{GE}) = \sum_{i=1}^{n} Cov(w_i R_i, R_{GE}) = Cov\left(\underbrace{\sum_{i=1}^{n} w_i R_i}_{R_M}, R_{GE}\right) \quad (4)$$

- Therefore, the reward-to-risk ratio for investments in GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} Cov(R_{GE}, R_M)} = \frac{E(R_{GE})}{Cov(R_{GE}, R_M)}$$

GE Example

- Reward-to-risk ratio for investment in market portfolio:

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(R_M)}{\sigma_M^2} \tag{5}$$

- Reward-to-risk ratios of GE and the market portfolio should be equal:

$$\frac{E(R_{GE})}{Cov(R_{GE}, R_M)} = \frac{E(R_M)}{\sigma_M^2} \tag{6}$$

rearrange

$$E(R_{GE}) = \frac{Cov(R_{GE}, R_M)}{\sigma_M^2} E(R_M) \tag{7}$$

GE Example, The CAPM Equation

$$E(r_{GE}) = r_f + \beta_{GE} [E(r_M) - r_f] \qquad (8)$$

Example

EXAMPLE 7.2

Expected Returns and Risk Premiums

Suppose the risk premium of the market portfolio is 9%, and we estimate the beta of Dell as $\beta_D = 1.3$. The risk premium predicted for the stock is therefore 1.3 times the market risk premium, or $1.3 \times 9\% = 11.7\%$. The expected rate of return on Dell is the risk-free rate plus the risk premium. For example, if the T-bill rate were 5%, the expected rate of return would be $5\% + 11.7\% = 16.7\%$ or, using Equation 7.2 directly,

$$E(r_D) = r_f + \beta_D[\text{Market risk premium}]$$
$$= 5\% + 1.3 \times 9\% = 16.7\%$$

If the estimate of the beta of Dell were only 1.2, the required risk premium for Dell would fall to 10.8%. Similarly, if the market risk premium were only 8% and $\beta_D = 1.3$, Dell's risk premium would be only 10.4%.

Expected Return-Beta Relationship

- CAPM holds for the overall portfolio because:

$$
\begin{aligned}
w_1 E(r_1) &= w_1 r_f + w_1 \beta_1 [E(r_M) - r_f] \\
+w_2 E(r_2) &= w_2 r_f + w_2 \beta_2 [E(r_M) - r_f] \\
&\vdots \\
+w_n E(r_n) &= w_n r_f + w_n \beta_n [E(r_M) - r_f]
\end{aligned}
$$

$$E(r_p) = r_f + \beta_p [E(r_M) - r_f]$$

Example

EXAMPLE 7.3

Portfolio Beta and Risk Premium

Consider the following portfolio:

Asset	Beta	Risk Premium	Portfolio Weight
Microsoft	1.2	9.0%	0.5
American Electric Power	0.8	6.0	0.3
Gold	0.0	0.0	0.2
Portfolio	0.84	?	1.0

If the market risk premium is 7.5%, the CAPM predicts that the risk premium on each stock is its beta times 7.5%, and the risk premium on the portfolio is .84 × 7.5% = 6.3%. This is the same result that is obtained by taking the weighted average of the risk premiums of the individual stocks. (Verify this for yourself.)

Concept check

CONCEPT
check 7.3

Suppose the risk premium on the market portfolio is estimated at 8% with a standard deviation of 22%. What is the risk premium on a portfolio invested 25% in GE with a beta of 1.15 and 75% in Dell with a beta of 1.25?

Figure The Security Market Line

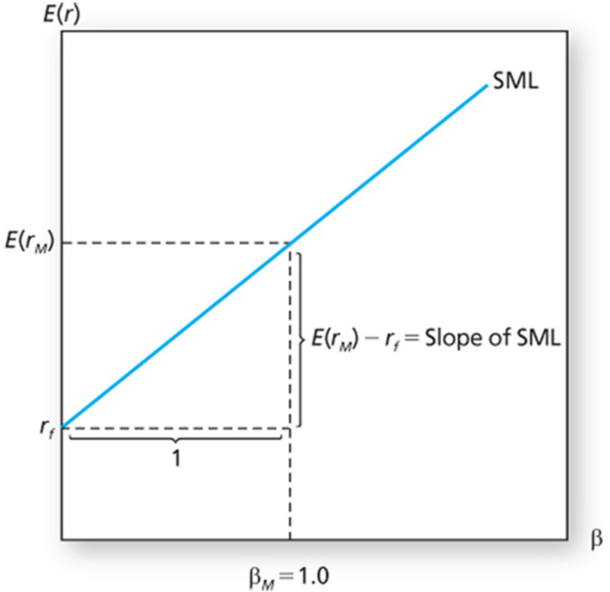
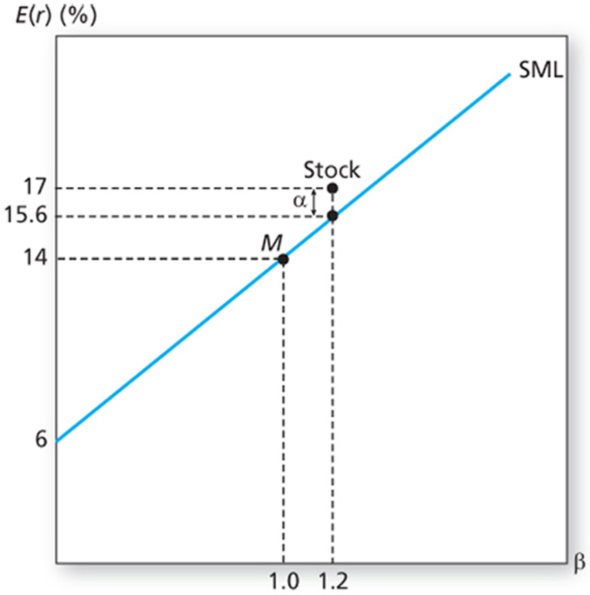


Figure 7.2 The SML and a Positive-Alpha Stock



Example

EXAMPLE 7.4

The Alpha of a Security

Suppose the return on the market is expected to be 14%, a stock has a beta of 1.2, and the T-bill rate is 6%. The SML would predict an expected return on the stock of

$$E(r) = r_f + \beta[E(r_M) - r_f]$$
$$= 6 + 1.2(14 - 6) = 15.6\%$$

If one believes the stock will provide instead a return of 17%, its implied alpha would be 1.4%, as shown in Figure 7.2. If instead the expected return were only 15%, the stock alpha would be negative, –.6%.

Example

EXAMPLE 7.5

The CAPM and Capital Budgeting

Suppose Silverado Springs Inc. is considering a new spring-water bottling plant. The business plan forecasts an internal rate of return of 14% on the investment. Research shows the beta of similar products is 1.3. Thus, if the risk-free rate is 4%, and the market risk premium is estimated at 8%, the hurdle rate for the project should be $4 + 1.3 \times 8 = 14.4\%$. Because the IRR is less than the risk-adjusted discount or hurdle rate, the project has a negative net present value and ought to be rejected.

Example

EXAMPLE 7.6

The CAPM and Regulation

Suppose shareholder equity invested in a utility is $100 million, and the equity beta is .6. If the T-bill rate is 6%, and the market risk premium is 8%, then a fair annual profit will be $6 + (.6 \times 8) = 10.8\%$ of $100 million, or $10.8 million. Since regulators accept the CAPM, they will allow the utility to set prices at a level expected to generate these profits.

Concept check

CONCEPT check 7.4

a. Stock XYZ has an expected return of 12% and $\beta = 1$. Stock ABC is expected to return 13% with a beta of 1.5. The market's expected return is 11% and $r_f = 5\%$. According to the CAPM, which stock is a better buy? What is the alpha of each stock? Plot the SML and the two stocks. Show the alphas of each on the graph.

b. The risk-free rate is 8% and the expected return on the market portfolio is 16%. A firm considers a project with an estimated beta of 1.3. What is the required rate of return on the project? If the IRR of the project is 19%, what is the project alpha?

CAPM and Single-Index Model

- To move from expected to realized returns, use the index model in excess return form:

$$R_i = \alpha_i + \beta_i R_M + e_i \tag{9}$$

- The index model beta coefficient is the same as the beta of the CAPM expected return-beta relationship.

Estimating the Index Model–Data

TABLE 7.1 Monthly return statistics: T-bills, S&P 500, and Google, January 2006–December 2010

Statistic (%)	T-bills	S&P 500	Google
Average rate of return	0.184	0.239	1.125
Average excess return	–	0.054	0.941
Standard deviation*	0.177	5.11	10.40
Geometric average	0.180	0.107	0.600
Cumulative total 5-year return	11.65	6.60	43.17
Gain Jan 2006–Oct 2007	9.04	27.45	70.42
Gain Nov 2007–May 2009	2.29	−38.87	−40.99
Gain June 2009–Dec 2010	0.10	36.83	42.36

*The rate on T-bills is known in advance, hence SD does not reflect risk.

http://somfin.gmu.edu/courses/fnan311/excel/fnan_311_data_ch07.xlsx

Figure 7.3 Returns

FIGURE 7.3 Returns for T-bills, S&P 500 Index, and Google stock. **Panel A:** monthly returns; **Panel B:** cumulative returns.

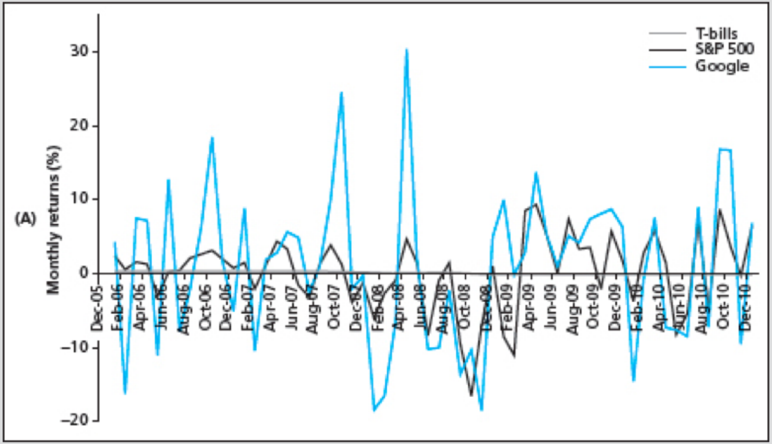


Figure 7.3 Panel B Cumulative Returns

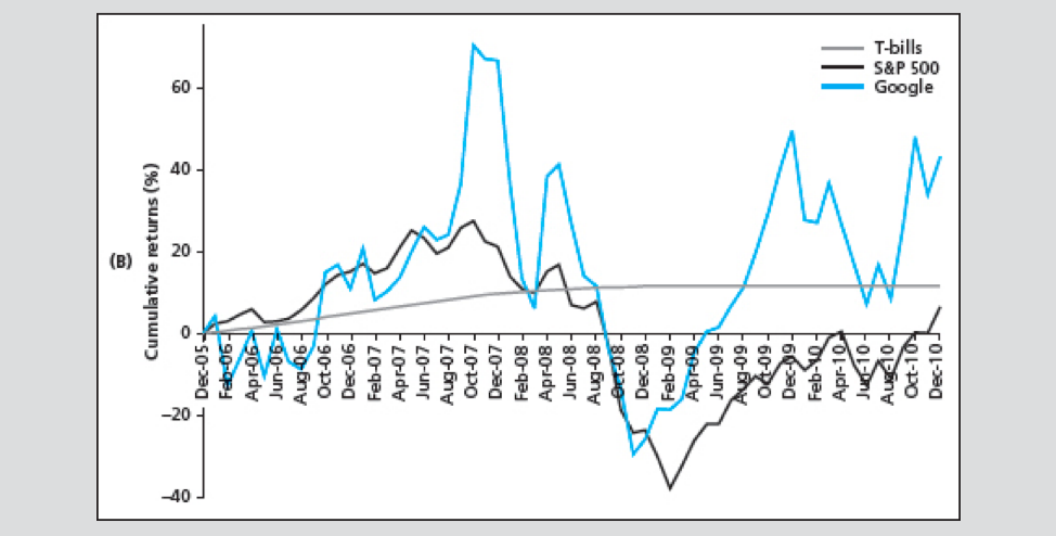


Figure 7.3 Panel B Cumulative Returns

FIGURE 7.4 Scatter diagram and security characteristic line for Google against the S&P 500, Jan 2006–Dec 2010

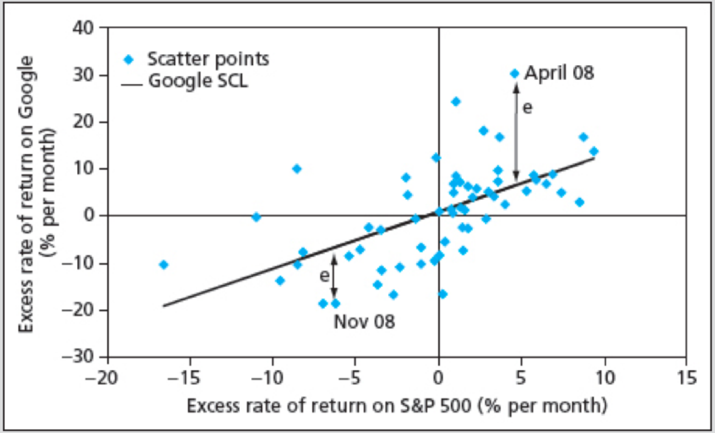


TABLE 7.2 Security characteristic line for Google (S&P 500 used as market index), January 2006–December 2010

Linear Regression

Regression Statistics	(This table produced by StatPlus patch for Mac Excel, which lacks the Data Analysis tool of Windows Excel)
R (correlation)	0.5914
R-square	0.3497
Adjusted *R*-square	0.3385
SE of regression	8.4585
Total number of observations	60

Regression equation: Google (excess return) = 0.8751 + 1.2031 * S&P 500 (excess return)

ANOVA

	df	*SS*	*MS*	*F*	*p*-level
Regression	1	2231.50	2231.50	31.19	0.0000
Residual	58	4149.65	71.55		
Total	59	6381.15			

	Coefficients	**Standard Error**	***t*-Statistic**	***p*-value**	**LCL**	**UCL**
Intercept	0.8751	1.0920	0.8013	0.4262	−1.7375	3.4877
S&P 500	1.2031	0.2154	5.5848	0.0000	0.6877	1.7185
t-Statistic (2%)	2.3924					

LCL—Lower confidence interval (95%)
UCL—Upper confidence interval (95%)

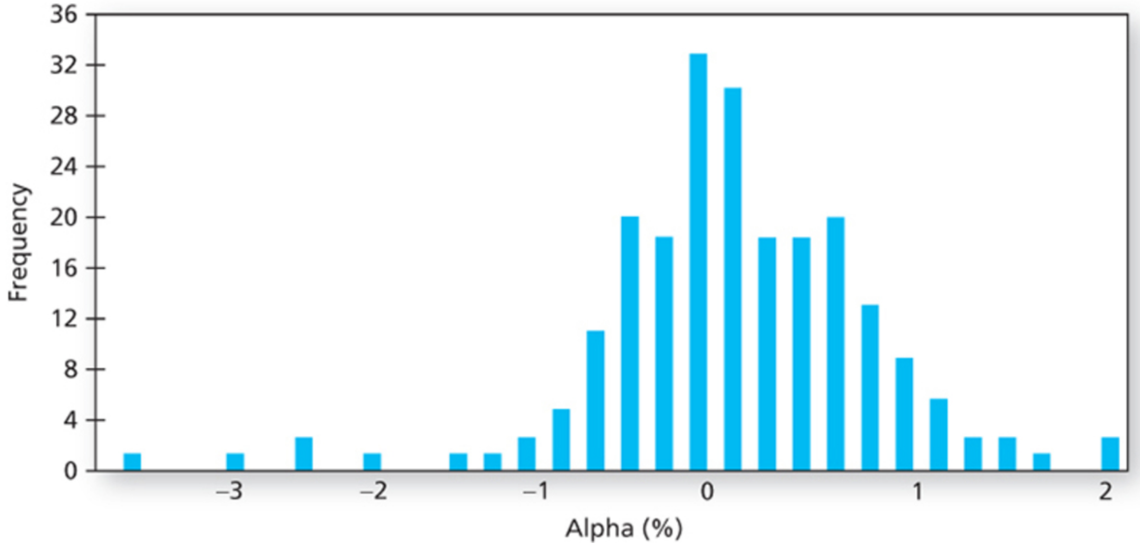
The CAPM and the Academic World

- The theoretical market portfolio impossible to observer (Roll 1977)
- Betas are unobservable, likely time varying
- Fama and French (1993) three factor model
- Generally the unconditional CAPM fails, some support for the conditional CAPM

The CAPM and the Investment Industry

- Relies on the single-index CAPM model
- Most investors don't beat the index portfolio. Hence it can be considered efficient and used:
  - for diversification
  - as a benchmark
  - to assess fair compensation for risk
  - for price regulation

Figure Estimates of Mutual Fund Alphas



Multifactor models and the CAPM

- Several systematic factors (sources or risk)
- In the case of two factors, the market and a Treasury-bond portfolio, the excess return R_{it} is:

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iTB} R_{TBt} + e_{it} \tag{10}$$

- The Expected Rate of return will have three components:
  1. The risk free rate
  2. The sensitivity (beta) to the market times the premium on the market
  3. The sensitivity (beta) to the TB portfolio times the premium on the portfolio

$$E(R_i) = r_f + \beta_{iM}[E(r_{Mt}) - r_f] + \beta_{iTB}[E(r_{TBt}) - r_f] \tag{11}$$

The above equation is expanding the SML in a third dimension.

EXAMPLE 7.8

A Two-Factor SML

Northeast Airlines has a market beta of 1.2 and a T-bond beta of .7. Suppose the risk premium of the market index is 6%, while that of the T-bond portfolio is 3%. Then the overall risk premium on Northeast stock is the sum of the risk premiums required as compensation for each source of systematic risk.

The risk premium attributable to market risk is the stock's exposure to that risk, 1.2, multiplied by the corresponding risk premium, 6%, or 1.2 × 6% = 7.2%. Similarly, the risk premium attributable to interest rate risk is .7 × 3% = 2.1%. The total risk premium is 7.2 + 2.1 = 9.3%. Therefore, if the risk-free rate is 4%, the expected return on the portfolio should be

4.0%	Risk-free rate
+ 7.2	+Risk premium for exposure to market risk
+ 2.1	+Risk premium for exposure to interest rate risk
13.3%	Total expected return

More concisely,

$$E(r) = 4\% + 1.2 \times 6\% + .7 \times 3\% = 13.3\%$$

Concept check

CONCEPT check 7.5

Suppose the risk premiums in Example 7.8 were $E(r_M) - r_f = 4\%$ and $E(r_{TB}) - r_f = 2\%$. What would be the equilibrium expected rate of return on Northeast Airlines?

Identifying Risk Factors

- Use theory, regression analysis, or statistical tools (e.g. principal components)
  - Theory states that risk factors should be related (or affect) consumption–assets which have low exposures (betas) to such risk factors would provide better returns when consumption is low than assets with high exposures, and they will have high prices (low premiums)
  - We need to show that such factors carry risk premiums, i.e.
- Some variables might help to explain returns but still might not carry a risk premium.
  - For example, securities of firms in the same industry may be highly correlated.
  - However, if this industry is a small part of the broad market, the industry risk can be diversified away.

The Fama-French Three Factors – Regression Analysis

- Multivariate regression:

$$r_{it} - r_f = \alpha_i + \beta_{iM}[r_{Mt} - r_f] + \beta_{i,HML}[r_{HML,t} - r_f] + \beta_{i,SMB}[r_{SMB,t} - r_f] + e_{it} \quad (12)$$

where

HML is a portfolio constructed by shorting the lowest book-to-market stocks and buying the highest book-to-market stocks

SMB is a portfolio constructed by shorting the largest market cap stocks and buying the smallest market cap stocks

Table 7.4 Regressing Google on Factor returns

TABLE 7.4 Regression statistics for alternative specifications:
1.A Single index with S&P 500 as market proxy
1.B Single index with broad market index (NYSE + NASDAQ + Amex)
2. Fama-French three-factor model (broad market + SMB + HML)

Monthly returns January 2006–December 2010

	Single Index Specification		
Estimate	S&P 500	Broad Market Index	FF 3-Factor Specification with Broad Market Index
Correlation coefficient	0.59	0.61	0.70
Adjusted *R*-Square	0.34	0.36	0.47
Residual SD = Regression SE (%)	8.46	8.33	7.61
Alpha = Intercept (%)	0.88 (1.09)	0.64 (1.08)	0.62 (0.99)
Market beta	1.20 (0.21)	1.16 (0.20)	1.51 (0.21)
SMB (size) beta	–	–	−0.20 (0.44)
HML (book to market) beta	–	–	−1.33 (0.37)

The Fama-French Three Factor Asset Pricing Model

$$E(r_i) = r_f + \beta_{iM}[E(r_M) - r_f] + \beta_{i,HML}[E(r_{HML}) - r_f] + \beta_{i,SMB}[E(r_{SMB}) - r_f] \tag{13}$$

Arbitrage Pricing Theory

- Arbitrage profit is a riskless profit with no effort and no money down.
- APT states that well functioning markets do not present arbitrage opportunities
- If a misalignment of prices occurs even a few investors can exploit and eliminate the arbitrage opportunities and correct mispricing very quickly.

Well-Diversified Portfolios and the APT

- Single index:

$$R_P = \alpha_P + \beta_P R_M + e_P$$

- Suppose $\alpha_P > 0$. We can hedge the systematic risk of P and create a zero beta portfolio:

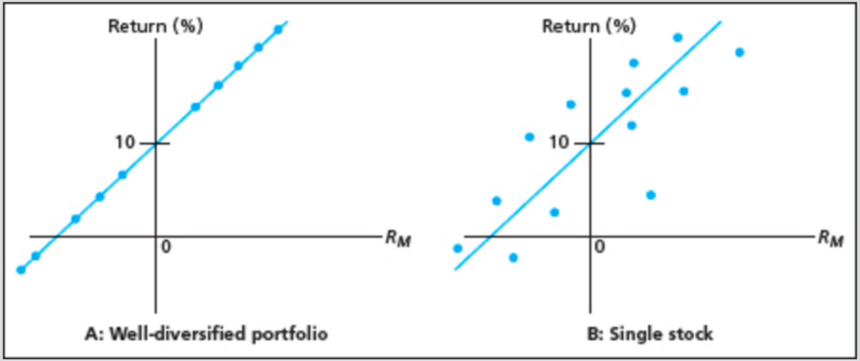
TABLE 7.5 Steps to convert a well-diversified portfolio into an arbitrage portfolio

Portfolio Weight*	In Asset	Contribution to Excess Return
$w_P = 1$	Portfolio P	$w_P(\alpha_P + \beta_P R_M + e_P) = \alpha_P + \beta_P R_M + e_P$
$w_M = -\beta_P$	Benchmark	$w_M R_M = -\beta_P R_M$
$w_f = \beta_P - 1$	Risk-free asset	$w_f \cdot 0 = 0$
$\Sigma w = 0$	Portfolio A	$\alpha_P + e_P$

Well-Diversified Portfolios and the APT

- a zero-investment, zero-beta, positive-alpha portfolio is not necessarily an arbitrage opportunity as $\sigma_e > 0$; true arbitrage implies no risk ($\sigma_e = 0$).
- If P is fully diversified, residual risk is minimal.

FIGURE 7.5 Security characteristic lines



EXAMPLE 7.9

Constructing an Arbitrage Portfolio

Suppose the benchmark, *M*, is the observed, broad market index that includes over 4,000 stocks (NYSE + NASDAQ + Amex). Imagine that on December 31, 2005, a portfolio manager possessed the following five-year predictions based on security and macro analyses:

1. The cumulative risk-free rate from rolling over T-bills over the next five years is estimated at 11.5%, an annual rate of 2.2%.

2. The cumulative return on the benchmark is estimated at 20%, an annual rate of 3.71%.

3. The S&P 500, which we will treat as portfolio *P*, is composed of large-capitalization stocks and is believed to be *overpriced*. Its expected cumulative return is forecast at 12%, an annual rate of 2.29%.

4. The S&P 500 beta against the benchmark is estimated at .95, which leads to the following calculation for its alpha: $2.29\% = 2.2\% + \alpha + .95(3.71 - 2.2)$; $\alpha = -1.34\%$ per year.

Because alpha is negative, we reverse the weights in Table 7.5 and set $w_P = -1$, $w_M = .95$, $w_f = .05$. The alpha on *A* is then positive: $\alpha_A = 1.34\%$.

Can we achieve a risk-free arbitrage portfolio?

- While the S&P 500 is highly diversified, is even this index sufficiently diversified to make A a risk-free arbitrage portfolio?

TABLE 7.6 Ten largest capitalization stocks in the S&P 500 portfolio and their weights (Dec. 31, 2009)

ExxonMobil	3.26	IBM	1.73
Microsoft	2.37	AT&T	1.67
Apple	1.91	JPMorgan Chase	1.65
Johnson & Johnson	1.79	GE	1.62
Procter & Gamble	1.78	Chevron	1.56
Total for 10 largest firms			19.34

Regressing S&P on broad market index (NYSE+NASDAQ+Amex)

TABLE 7.7 Regression statistics of the S&P 500 portfolio on the benchmark portfolio, January 2006–December 2010

Linear Regression

Regression Statistics		
R	0.9933	
R-square	0.9866	
Adjusted *R*-square	0.9864	**Annualized**
Regression SE	0.5968	2.067
Total number of observations	60	

S&P 500 = −0.1909 + 0.9337* Benchmark

	Coefficients	Standard Error	*t*-stat	*p*-level
Intercept	−0.1909	0.0771	−2.4752	0.0163
Benchmark	0.9337	0.0143	65.3434	0.0000

Summary of Regression results

- $\alpha_{S\&P} = -0.19\%$ or -2.27% per year
- $\beta_{S\&P,M} = 0.93$
- The above estimates are close to the 'prediction' in Ex. 7.9.
- σ_e is 2.07% per year – is this small-enough to determine that the portfolio is well-diversified?
- To exexute an arbitrage strategie, you also need to invest in T-bills. Is this completely risk-free in reality?

SD of T-bill historical returns

TABLE 7.8 Annual standard deviation of the real, inflation, and nominal rates

Period	Real Rate	Inflation Rate	Nominal Rate
1/1/2006–12/31/2010	1.46	1.46	0.61
1/1/1996–12/31/2000	0.57	0.54	0.17
1/1/1986–12/31/1990	0.86	0.83	0.37

- SD of the real rate from rolling over bills is in the range of .5%1.5% per year.
- Need to consider this risk as well when constructing arbitrage strategy.

Marginal risk of S&P Portfolio

SD of Real Rate on Initial Portfolio	SD of Total Portfolio	Marginal Risk
0.5% (T-bills)	$(.005^2 + .0207^2)^{1/2} = 2.13\%$	1.63%
1.0 (T-bills)	2.30	1.30
1.5 (T-bills)	2.56	1.06
20.0 (benchmark)	20.11	0.11

- when the arbitrage portfolio is added to the risky benchmark portfolio, the marginal increase in overall standard deviation is minimal.

Derifing the APT

- If a zero investment portfolio has a significant alpha, this will result in large scale transactions:
  - purchase positive alpha portfolios
  - short negative alpha portfolios
- These transactions will drive the alphas to zero:

$$r_P = r_f + \beta_P(r_M - r_f) + e_P \tag{14}$$

Taking expectations:

$$E(r_P) = r_f + \beta_P[E(r_M) - r_f] \tag{15}$$

The APT and The CAPM

- APT
  - Assumes a well-diversified portfolio, but residual risk is still a factor.
  - Does not assume investors are mean-variance optimizers.
  - Uses an observable, market index
  - Reveals arbitrage opportunities
- CAPM
  - Model is based on an inherently unobservable market portfolio.
  - Rests on mean-variance efficiency. The actions of many small investors restore CAPM equilibrium.

Multifactor Generalization of the APT and CAPM

- Expanding the single-factor model to a two-factor model:

$$R_i = \alpha_i + \beta_{i1} R_{M1} + \beta_{i2} R_{M2} + e_i \qquad (16)$$

where R_{M1} and R_{M2} are the excess returns on portfolios that represent the two systematic factors.

EXAMPLE 7.10

Multifactor SML

Suppose the two-factor portfolios, here called portfolios 1 and 2, have expected returns $E(r_1)$ = 10% and $E(r_2)$ = 12%. Suppose further that the risk-free rate is 4%. The risk premium on the first factor portfolio is therefore 6%, while that on the second factor portfolio is 8%.

Now consider an arbitrary well-diversified portfolio (P), with beta on the first factor, $\beta_{P1} = .5$, and on the second factor, $\beta_{P2} = .75$. The multifactor APT states that the portfolio risk premium must equal the sum of the risk premiums required as compensation to investors for each source of systematic risk. The risk premium attributable to risk factor 1 is the portfolio's exposure to factor 1, β_{P1}, times the risk premium earned on the first factor portfolio, $E(r_1) - r_f$. Therefore, the portion of portfolio P's risk premium that is compensation for its exposure to the first risk factor is $\beta_{P1}[E(r_1) - r_f] = .5(10\% - 4\%) = 3\%$, while the risk premium attributable to risk factor 2 is $\beta_{P2}[E(r_2) - r_f] = .75(12\% - 4\%) = 6\%$. The total risk premium on the portfolio, therefore, should be 3 + 6 = 9%, and the total return on the portfolio should be 13%.

4%	Risk-free rate
+ 3%	Risk premium for exposure to factor 1
+ 6%	Risk premium for exposure to factor 2
13%	Total expected return

Constructing arbitrage portfolio with two factors

- Suppose portfolio P of Example 7.10 actually has an expected excess return of 11% and therefore a positive alpha of 2%.

TABLE 7.9 Constructing an arbitrage portfolio with two systematic factors

Portfolio Weight	In Asset	Contribution to Excess Return
1	Portfolio P	$\alpha_P + \beta_{P1}R_1 + \beta_{P2}R_2 + e_P = 11\% + e_P$
$-\beta_{P1} = -0.5$	Factor portfolio 1	$\beta_{P1}R_1 = -.5 \times 6\% = -3\%$
$-\beta_{P2} = -0.75$	Factor portfolio 2	$\beta_{P2}R_2 = -.75 \times 8\% = -6\%$
$\beta_{P1} + \beta_{P1} - 1 = 0.25$	Risk-free asset	0
Total = 1	Portfolio A	$\alpha_P + e_P = 2\% + e_P$

- Because P is well diversified, eP must be small, and the excess return on the zero-investment, zero-beta portfolio A is just $\alpha_P = 2\%$.

Concept check

CONCEPT *check* **7.6**

Using the factor portfolios of **Example 7.10**, find the fair rate of return on a security with $\beta_1 = .2$ and $\beta_2 = 1.4$.

Estimating the index model

	A	B	C	D	E	F	G	H	I	J
3		----------------Rates of Return------------					---------Excess rates of return---------			
4	Month	Ford	Honda	Toyota	S&P	T-bills	Ford	Honda	Toyota	S&P
5	Dec-08	-18.34	23.02	-2.95	-8.31	0.09	-18.43	22.93	-3.04	-8.40
6	Nov-08	-14.87	-25.44	3.71	0.97	0.02	-14.89	-25.46	3.69	0.95
7	Oct-08	22.83	-26.04	-17.07	-7.04	0.08	22.75	-26.12	-17.15	-7.12
8	Sep-08	-57.88	-13.77	-11.32	-16.67	0.15	-58.03	-13.92	-11.47	-16.82
9	Aug-08	16.59	-29.61	-4.23	-9.54	0.12	16.47	-29.73	-4.35	-9.66
10	Jul-08	-7.08	-4.92	4.11	1.40	0.15	-7.23	-5.07	3.96	1.25
11	Jun-08	-0.21	-11.01	-8.46	-1.07	0.17	-0.38	-11.18	-8.63	-1.24
12	May-08	-29.26	-15.56	-7.89	-8.52	0.17	-29.43	-15.73	-8.06	-8.69
13	Apr-08	-17.68	-6.48	0.54	1.34	0.17	-17.85	-6.65	0.37	1.17
14	Mar-08	44.41	8.78	0.60	4.60	0.17	44.24	8.61	0.43	4.43
15	Feb-08	-12.40	-9.60	-7.06	-1.03	0.13	-12.53	-9.73	-7.19	-1.16
16	Jan-08	-1.66	-3.13	0.21	-2.79	0.21	-1.87	-3.34	0.00	-3.00
17	Dec-07	-1.34	11.18	2.03	-6.31	0.27	-1.61	10.91	1.76	-6.58
18	Nov-07	-10.39	-5.59	-5.58	-1.47	0.34	-10.73	-5.93	-5.92	-1.81
19	Oct-07	-15.33	-6.86	-1.74	-4.19	0.32	-15.65	-7.18	-2.06	-4.51
20	Sep-07	4.48	-0.25	-2.07	1.03	0.32	4.16	-0.57	-2.39	0.71
21	Aug-07	8.71	8.79	1.02	3.46	0.42	8.29	8.37	0.60	3.04
22	Jul-07	-8.23	-4.78	-4.10	0.88	0.4	-8.63	-5.18	-4.50	0.48
23	Jun-07	-9.66	-6.32	-4.17	-3.53	0.4	-10.06	-6.72	-4.57	-3.93
24	May-07	12.95	-0.24	4.24	-1.87	0.41	12.54	-0.65	3.83	-2.28
25	Apr-07	3.73	1.87	-0.54	2.95	0.44	3.29	1.43	-0.98	2.51
26	Mar-07	1.90	-0.08	-5.26	3.99	0.43	1.47	-0.51	-5.69	3.56

http://somfin.gmu.edu/courses/fnan311/excel/ss7p1.xlsx

Excel Questions

Excel Questions

1. What were the betas of Ford, Toyota, and Honda?

2. In light of each firm's exposure to the financial crisis in 2008-2009, does the value for Ford compared to Honda and Toyota make sense to you?