Chapter 18, Portfolio Performance Evaluation

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Chapter Overview

Introduction

- If markets are efficient, investors must be able to measure asset management performance
- Two common ways to measure average portfolio return:
  1. Time-weighted returns
  2. Dollar-weighted returns
- Returns must be adjusted for risk.

Dollar- and Time-Weighted Average Returns

- Time-weighted returns
  - The geometric average is a time-weighted average.
  - Each period's return has equal weight.

$$(1 + r_G)^n = (1 + r_1)(1 + r_2) \dots (1 + r_n)$$

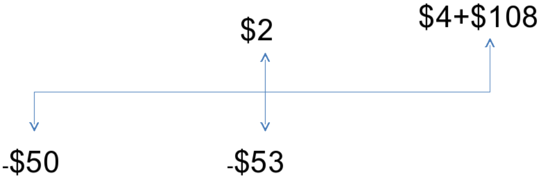
Dollar- and Time-Weighted Average Returns

- Dollar-weighted returns
  - Internal rate of return considering the cash flow from or to investment
  - Returns are weighted by the amount invested in each period:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

Example of Multiperiod Returns

Time	Outlay
0	$50 to purchase first share
1	$53 to purchase second share a year later
	Proceeds
1	$2 dividend from initially purchased share
2	$4 dividend from the 2 shares held in the second year, plus $108 received from selling both shares at $54 each



- Dollar-weighted return:

$$50 + \frac{53}{1+r} = \frac{2}{1+r} + \frac{112}{(1+r)^2} \qquad r = 7.117\%$$

Time-Weighted Return

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%$$
$$r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$
$$r_G = [(1.1)(1.0566)]^{1/2} - 1 = 7.81\%$$

- The dollar-weighted average is less than the time-weighted average in this example because more money is invested in year two, when the return was lower.

Time-Weighted Return

- Households should maintain a spreadsheet of time-dated cash flows (in and out) to determine the effective rate of return for any given period.
- Examples include:
  - IRA, 401(k), 529

Passive vs Active Management

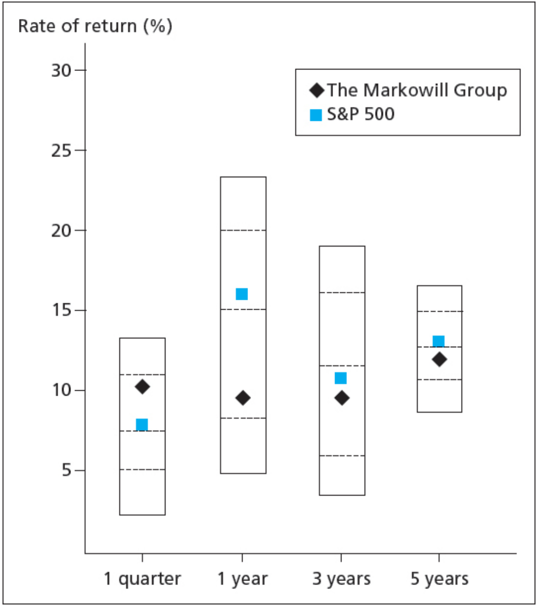
- Passive Management
  - Diversified portfolio with no security mispricing identification
  - Cash
    - Virtually risk-free money market securities
  - Choose an allocation between risky portfolio and cash and hold it long-term.
- Active Management
  - Forecasting broad markets and/or identifying mispriced securities to achieve higher returns
  - Market Timing
    - Relative performance drives fund movement between risky portfolio and cash

Adjusting Returns for Risk

- The simplest and most popular way to adjust returns for risk is to compare the portfolio's return with the returns on a comparison universe.
- The comparison universe is a benchmark composed of a group of funds or portfolios with similar risk characteristics, such as growth stock funds or high-yield bond funds.

Figure 18.1 Universe Comparison

Figure 18.1. The chart summarizes performance rankings over four periods: 1 quarter, 1 year, 3 years, and 5 years. The top and bottom lines of each box are drawn at the rate of return of the 95th and 5th percentile managers. The three dashed lines correspond to the rates of return of the 75th, 50th (median), and 25th percentile managers.



Risk Adjusted Performance

- To measure abnormal performance, we must measure normal performance.
  - Single index model can be used:

$$R_{Pt} = \beta_P R_{Mt} + \alpha_P + e_{Pt} \tag{1}$$

  - $\alpha_P + e_{Pt}$ is extra performance in time (month) t
- The expected excess return of the portfolio over some evaluation period is:

$$E(R_{Pt}) = \beta_P E(R_{Mt}) + \alpha_P \tag{2}$$

- We measure expected returns over the period (unfortunately, with sampling error) by average return.

Risk Adjusted Performance

- Variance of market-driven return component:

$$Var(\beta_P R_{Mt}) = \beta_P^2 \sigma_M^2 \tag{3}$$

- Variance of return of P:

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma_e^2 \tag{4}$$

Risk Adjusted Performance: Prepare Statistics

- Steps in preparing the statistics that are used for performance evaluation
  1. Obtain the time series of R_{Pt} for portfolio P, and R_{Mt} for the benchmark M.
  2. Compute the arithmetic averages of the series, $\bar{R}_{Pt}$ and, $\bar{R}_{Mt}$. These proxy for $E(R_{Pt})$ and $E(R_{Mt})$.
  3. Compute σ_P and σ_M.
  4. Run a regression of R_{Pt} on R_{Mt} to obtain estimates of P's beta, alpha, residual SD, and correlation with the benchmark.
     - Are the coefficients significant?
     - Does the sample have a sufficient number of observations?
  5. The regression intercept is P's alpha
  6. The standard error, or residual standard deviation, of the regression, is σ_e

Table 18.1 Performance of two managed portfolios

TABLE 18.1 Performance of two managed portfolios, *P* and *Q*, the benchmark portfolio, *M*, and cash equivalents

	Portfolio *P*	Portfolio *Q*	Benchmark	Cash
Average return	13.6	9.5	10.4	4
Average excess return (%)	9.60	5.50	6.37	0
Standard deviation (%)	24.1	18.0	18.5	0
Beta (pure number)	1.25	0.50	1.0	0
Alpha (%)	1.6	2.3	0	0
Residual SD (%)	6.79	15.44	0	0
Correlation with benchmark	0.96	0.51	1	0
Sharpe ratio	0.398	0.306	0.344	0
M-square (%)	1.00	−0.72	0	0
Treynor measure	7.68	11.00	6	0
Information ratio	0.24	0.15	0	0

Risk Adjusted Performance: Sharpe

- Sharpe (reward-to-variability) ratio:

$$S = \frac{\bar{r}_P - \bar{r}_f}{\bar{\sigma}_P}$$

where

$$\begin{aligned} \bar{r}_P &= \text{Average return on the portfolio} \\ \bar{r}_f &= \text{Average risk-free rate} \\ \bar{\sigma}_P &= \text{standard deviation of portfolio return} \end{aligned}$$

- Interpretation: the incremental return an investor may expect for every increase of 1% of standard deviation, i.e. compensation per unit or risk
- Used when choosing among competing portfolios that will not be mixed.

The M^2 Measure

- Can we transform P to an equivalent portfolio with the same standard deviation as the benchmark, σ_M, without affecting its Sharpe ratio?
- We form P^* by choosing $w = \frac{\sigma_M}{\sigma_P}$ which makes $\sigma_{P^*} = w\sigma_P = \sigma_M$. The risk premium of P^* is:

$$\bar{R}_{P^*} = w\bar{R}_P = \frac{\sigma_M}{\sigma_P}\bar{R}_P = \sigma_M S_P$$

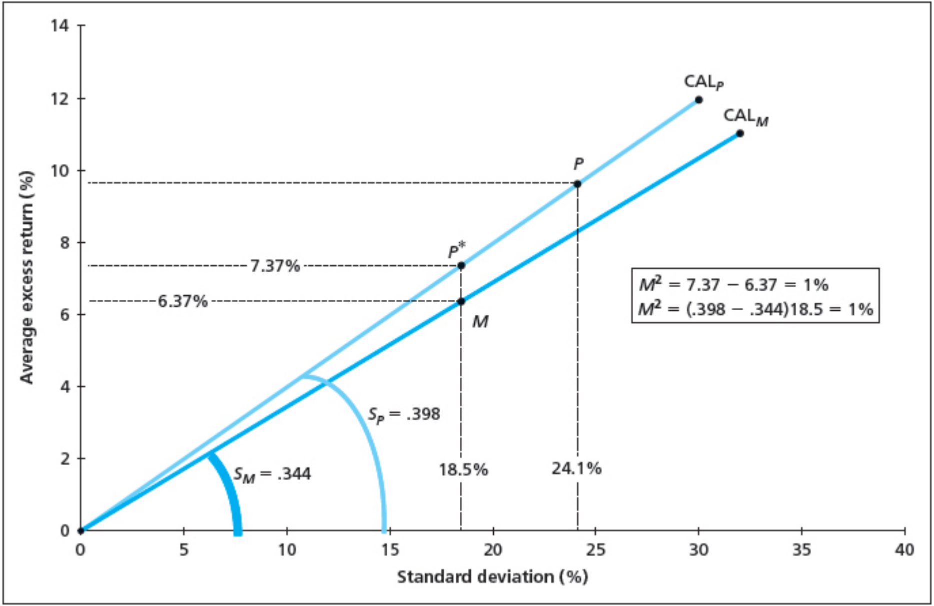
similarly, $\bar{R}_M$ is

$$\bar{R}_M = \frac{\sigma_M}{\sigma_M}\bar{R}_M = \sigma_M S_M$$

- M^2 is the difference between the premium on the adjusted portfolio P^* and the benchmark:

$$M^2 = \bar{R}_{P^*} - \bar{R}_M = \sigma_M(S_P - S_M) \tag{5}$$

Figure 18.2. The M^2 measure. We move down the capital allocation line CAL_P corresponding to portfolio P (by mixing P with T-bills) until we reduce σ_{P*} to match that of the market index.



Risk Adjusted Performance: Treynor

$$\text{Treynor Measure} = \frac{\bar{r}_P - \bar{r}_f}{\beta_P}$$

where

$$\bar{r}_P = \text{Average return on the portfolio}$$
$$\bar{r}_f = \text{Average risk-free rate}$$
$$\beta_P = \text{weighted average beta for portfolio}$$

- Used to evaluate a portfolio that is part of a larger portfolio with different managers

Treynor vs. Sharpe

	Treynor (T_p)	Sharpe* (S_p)
Relation to alpha	$\frac{E(r_p) - r_f}{\beta_p} = \frac{\alpha_p}{\beta_p} + T_M$	$\frac{E(r_p) - r_f}{\sigma_p} = \frac{\alpha_p}{\sigma_p} + \rho S_M$
Deviation from market performance	$T_p^2 = T_p - T_M = \frac{\alpha_p}{\beta_p}$	$S_p - S_M = \frac{\alpha_p}{\sigma_p} - (1 - \rho) S_M$

*ρ denotes the correlation coefficient between portfolio P and the market, and is less than 1.

Information Ratio

Ratio of alpha to standard deviation of diversifiable risk

$$\text{information Ratio} = \frac{\alpha_P}{\sigma(e_P)}$$

where

$$\alpha_P = \text{Alpha for the portfolio}$$
$$\bar{\sigma}_P = \text{standard deviation of unexpected portfolio return}$$

- The information ratio divides portfolio alpha by its nonsystematic risk.
- Nonsystematic risk could, in theory, be eliminated by diversification.

Information Ratio

- Active managers are extensively using this measure.
- Useful when considering adding a position in an actively managed portfolio to an already existing passive portfolio.
- The combined (optimized) portfolio's Sharpe ration is then:

$$S_O = \sqrt{S_M^2 + \left(\frac{\alpha_P}{\sigma_P}\right)^2} \tag{6}$$

Table Comparing Measures

Performance Measure	Definition	Application
Sharpe	$\frac{\text{Excess return}}{\text{Standard deviation}}$	When choosing among portfolios competing for the overall risky portfolio
Treynor	$\frac{\text{Excess return}}{\text{Beta}}$	When ranking many portfolios that will be mixed to form the overall risky portfolio
Information ratio	$\frac{\text{Alpha}}{\text{Residual standard deviation}}$	When evaluating a portfolio to be mixed with the benchmark portfolio

Risk Adjusted Performance: Jensen

Jensen's Alpha

$$\alpha_P = \bar{r}_P - [\bar{r}_f + \beta_P(\bar{r}_M - \bar{r}_f)]$$

where

$$\alpha_P = \text{Alpha for the portfolio}$$
$$\bar{r}_P = \text{Average return on the portfolio}$$
$$\bar{r}_f = \text{Average risk-free rate}$$
$$\beta_P = \text{weighted average beta for portfolio}$$
$$\bar{r}_M = \text{Average return on the market index portfolio}$$

- Measure of abnormal returns
- Must establish statistical significance via regression

Risk Adjusted Performance: Jensen

- When short sales are allowed, a negative alpha is just as good as a positive one. The short position can also reduce portfolio beta while it provides abnormal return.
- If short sales are not allowed then a negative alpha must be ignored.
- Since we judge ex-post (after the fact), a negative realized alpha means below-average performance.

Jensen's alpha vs. Treynor and Sharpe

- Relationship of Jensen's measure and the Sharpe ratio:

$$S_P = \frac{\bar{R}_P}{\sigma_P} = \frac{\beta_P \bar{R}_M}{\sigma_P} + \frac{\alpha_P}{\sigma_P}$$
$$\beta_P = \frac{Cov(R_p, R_M}{\sigma_M^2} = \rho \frac{\sigma_P}{\sigma_M}$$
$$S_P = S_M \times \rho + \frac{\alpha_P}{\sigma_P} \tag{7}$$
$$S_P - S_M = S_M \times (\rho - 1) + \frac{\alpha_P}{\sigma_P}$$

- Thus positive alpha is not a sufficient condition for a managed portfolio to offer a higher Sharpe measure than the passive benchmark.
- A positive alpha is necessary to obtain a higher Sharpe ratio than the benchmark's sharpe ratio, because $S_M(\rho 1)$ is negative.

Jensen's alpha vs. Treynor and Sharpe

- Relationship of Jensen's measure and the Treynor measure:

$$
\begin{aligned}
T_P = \frac{\bar{R}_P}{\beta_P} = \frac{\beta_P \bar{R}_M}{\beta_P} + \frac{\alpha_P}{\beta_P} = \bar{R}_M + \frac{\alpha_P}{\beta_P} \\
\beta_M = 1 \qquad T_M = \bar{R}_M \\
T_P - T_M = \frac{\alpha_P}{\beta_P}
\end{aligned}
\tag{8}
$$

- Thus positive alpha is not a sufficient condition for a managed portfolio to offer a higher Treynor measure.
- We need to know β as well.
- We can be sure, though, that a negative alpha indicates inferior performance by all performance measures.

Concept check

CONCEPT check 18.1

Consider the following data for a particular sample period when returns were high:

	Portfolio *P*	Market *M*
Average return	35%	28%
Beta	1.2	1.0
Standard deviation	42%	30%

Calculate alpha and the three performance measures for portfolio *P* and the market. The T-bill rate during the period was 6%. By which measures did portfolio *P* outperform the market?

Alpha Capture and Alpha Transport

- Alpha capture:
  - Many hedge funds want positive alpha with zero beta, seeking abnormal returns without taking a stance on the direction of the market.
  - Solution: hedge out the market exposure of the portfolio by selling either the stock index or stock-index futures
  - Alpha capture is implementing a long-short market neutral position while maintaining positive alpha
- Alpha Transport:
  - With the captured alpha, you can establish any desired sensitivity to particular market sectors using index products such as ETFs.
  - This way you transfer alpha from the sector where you find it to the market sector in which you seek exposure

EXAMPLE 18.1

Alpha Capture and Transport

Zeta, a portfolio manager, established a positive-alpha portfolio *P* with a positive exposure to the market index: $\beta_{PM} = 1.3$. Now she wishes to transfer the alpha. Her objective is a portfolio that is market neutral but with positive exposure to the health care sector. In other words, she wants to "transport" her positive-alpha portfolio from a broad market exposure to a narrow health care exposure, a sector she believes will outperform. Her goal is a zero-net-investment position with a beta of zero on the market index but with a beta of .5 on a health care sector index.

We call Zeta's final portfolio *Z,* which will be constructed from positions in the original positive-alpha portfolio *P,* the market index portfolio *M,* the health care index portfolio *H,* and the risk-free asset *F.* Zeta will first isolate alpha by neutralizing *P*'s market beta. She will then use a health care sector index portfolio to establish her desired exposure to health care. In the end, she wants her final portfolio *Z* to have a zero beta on the broad market, $\beta_{ZM} = 0$, and a beta of .5 on health care, $\beta_{ZH} = .5$.

Zeta's statistical analysis implies that a health care exchange-traded fund, XLV, has a market beta, $\beta_{XLV} = .9$. Therefore, as she establishes exposure to the health care portfolio, she will also take on market exposure, and this too must be hedged away. Therefore, as Table 18.2 shows, she must take a position in the market index sufficiently large to offset the beta of portfolio *P* as well as the additional market exposure created by her position in the health care ETF. The hedging strategy that creates pure exposure to the health care sector is similar to the hedging of factor exposures that we encountered in the discussion of the arbitrage pricing theory (see Tables 7.5 and 7.9).[5]

TABLE 18.2 Alpha capture and transfer to the health care sector

Portfolio Weight*	In Asset	Contribution to Excess Returns
$w_P = 1$	P	$w_P(\alpha_P + \beta_{PM} R_M + e_P) = \alpha_P + 1.3 R_M + e_P$
$w_{XLV} = .5$	XLV	$w_{XLV} R_{XLV} = .5(.9 R_M + e_{XLV}) = .45 R_M + .5e_{XLV}$
$w_M = -\beta_P - .5\beta_{XLV} = -1.75$	M	$w_M R_M = -1.75 R_M$
$w_F = -1 - .5 + 1.75$	Risk-free	0
0	Portfolio Z	$\alpha_P + e_P + .5e_{XLV}$

Evaluation with Multi-Index model

- Using the Fama-French three factor model instead of the CAPM

$$R_{Pt} = \beta_P R_{Mt} + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \alpha_P + e_{Pt} \tag{9}$$

$$\bar{R}_{Pt} = \beta_P \bar{R}_{Mt} + \beta_{SMB} \bar{r}_{SMB,t} + \beta_{HML} \bar{r}_{HML,t} + \alpha_P \tag{10}$$

Example Excel

The Excel model "Performance Measures" calculates all of the performance measures discussed in this chapter. The model available on our website is built to allow you to compare eight different portfolios and to rank them on all measures discussed in this chapter.

eXcel

Please visit us at
www.mhhe.com/bkm

	A	B	C	D	E	F	G	H	I	J	K
1	Performance Measurement										
2											
3											
4											
5		Average	Standard	Beta	Unsystematic	Sharpe	Treynor	Jensen	M^2	T^2	Appraisal
6	Fund	Return	Deviation	Coefficient	Risk	Ratio	Measure	Alpha	Measure	Measure	Ratio
7	Alpha	.2800	.2700	1.7000	.0500	0.8148	.1294	-.0180	-.0015	-.0106	-0.3600
8	Omega	.3100	.2600	1.6200	.0600	0.9615	.1543	.0232	.0235	.0143	0.3867
9	Omicron	.2200	.2100	0.8500	.0200	0.7619	.1882	.0410	-.0105	.0482	2.0500
10	Millennium	.4000	.3300	2.5000	.2700	1.0303	.1360	-.0100	.0352	-.0040	-0.0370
11	Big Value	.1500	.1300	0.9000	.0300	0.6923	.1000	-.0360	-.0223	-.0400	-1.2000
12	Momentum Watcher	.2900	.2400	1.4000	.1600	0.9583	.1643	.0340	.0229	-.0243	0.2125
13	Big Potential	.1500	.1100	0.5500	.0150	0.8182	.1636	.0130	-.0009	.0236	0.8667
14	S&P Index Return	.2000	.1700	1.0000	.0000	0.8235	.1400	.0000	.0000	.0000	0.0000
15	T-Bill Return	.06		0							

http://somfin.gmu.edu/courses/fnan311/excel/Chapter_18_Performance_Measures.xlsx

Which Measure is Appropriate?

- It depends on investment assumptions
  1. If P is not diversified, then use the Sharpe measure as it measures reward to risk.
  2. If the P is diversified, non-systematic risk is negligible and the appropriate metric is Treynor's, measuring excess return to beta.

Table with Data on Portfolios P and Q

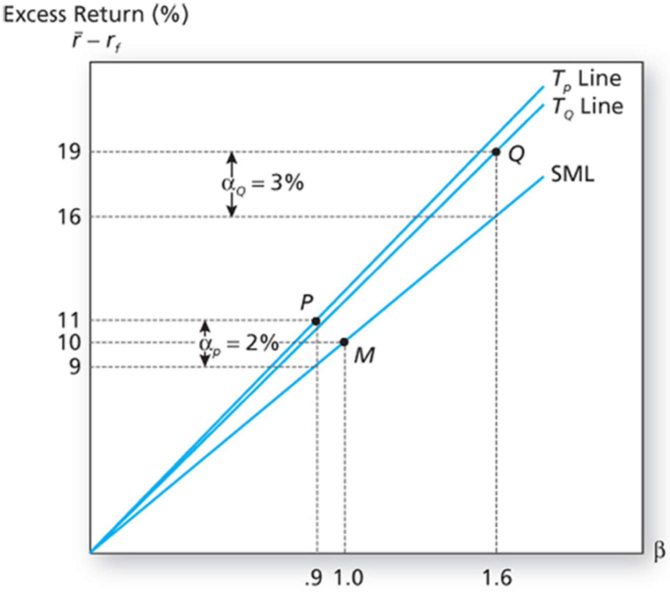
	Portfolio P	Portfolio Q	Market
Beta	.90	1.60	1.0
Excess return $(\bar{r} - \bar{r}_f)$	11%	19%	10%
Alpha*	2%	3%	0

*Alpha = Excess return − (Beta × Market excess return)
$= (r - r_f) - \beta(r_M - r_f) = r - [r_f + \beta(r_M - r_f)]$

- Portfolio Q has higher alpha.
- We focus on beta because P and Q are two of many fund portfolios.

Figure Treynor's measure

The Figure shows the T-lines (combinations of portfolios with T-bills) for portfolios P and Q. P has a steeper T-line; despite its lower alpha, P is a better portfolio after all. For any given beta, a mixture of P with T-bills will give a better alpha than a mixture of Q with T-bills.



Style Analysis

- Complex method of performance evaluation introduced by William Sharpe
- Recent studies of mutual fund performance show $> 90\%$ of return variation can be explained by funds' allocations to T-bills, stocks, and bonds
- Sharpe considered 12 asset class (style) portfolios.
  - regress fund returns on indexes
  - constrain coefficients: $\alpha = 0$, $\beta_i > 0$ and $\sum_i \beta_i = 100\%$
  - residuals are not constrained to sum to zero—sum equals the total return from security selection.

Table 18.3 Sharpe's Style portfolios for Magellan Fund

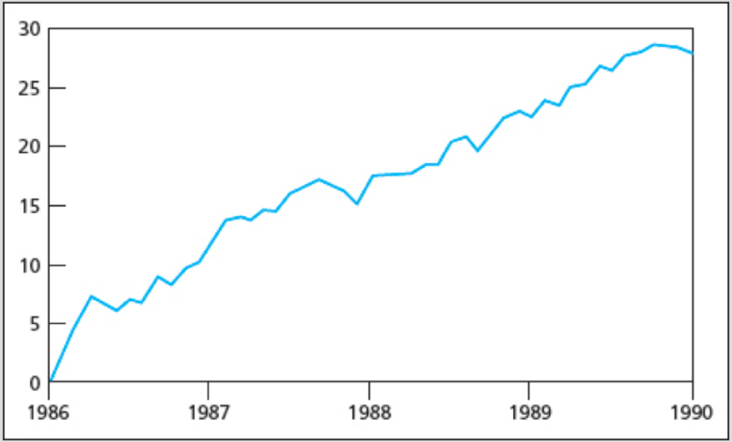
	Regression Coefficient*
Bills	0
Intermediate bonds	0
Long-term bonds	0
Corporate bonds	0
Mortgages	0
Value stocks	0
Growth stocks	47
Medium-cap stocks	31
Small stocks	18
Foreign stocks	0
European stocks	4
Japanese stocks	0
Total	100
R-squared	97.3%

Style Analysis

- The proportion of return variability not explained by asset allocation can be attributed to security selection within asset classes.
  - For Magellan, this was $100 - 97.3 = 2.7\%$.
  - The cumulative effect of the residuals (return difference vs style benchmark) indicates superior stock selection (Fig 18.3)
  - Magellan's consistently positive residual returns is rather uncommon (Fig 18.5).

Figure 18.3 Cumulative effect of residuals

FIGURE 18.3 Fidelity Magellan Fund cumulative return difference: Fund versus style benchmark



Source: William F. Sharpe, "Asset Allocation: Management Style and Performance Evaluation," *Journal of Portfolio Management,* Winter 1992, pp. 7–19. Figure 17, p. 18. Used with permission of Institutional Investor, Inc., **www.iijournals.com**.

Figure 18.4 Cumulative return difference vs S&P benchmark

FIGURE 18.4 Fidelity Magellan Fund cumulative return difference: Fund versus S&P 500

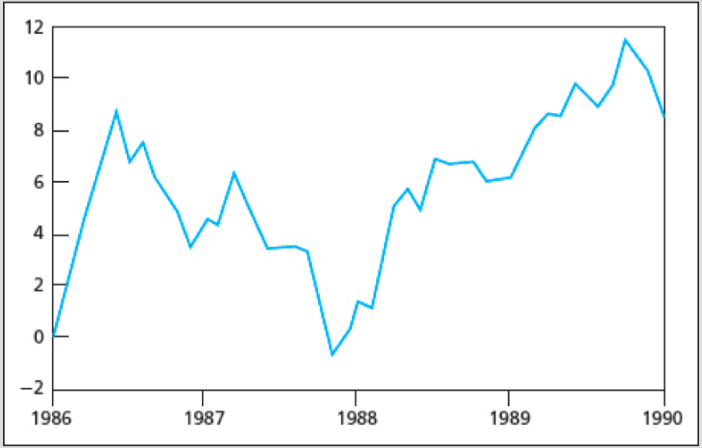


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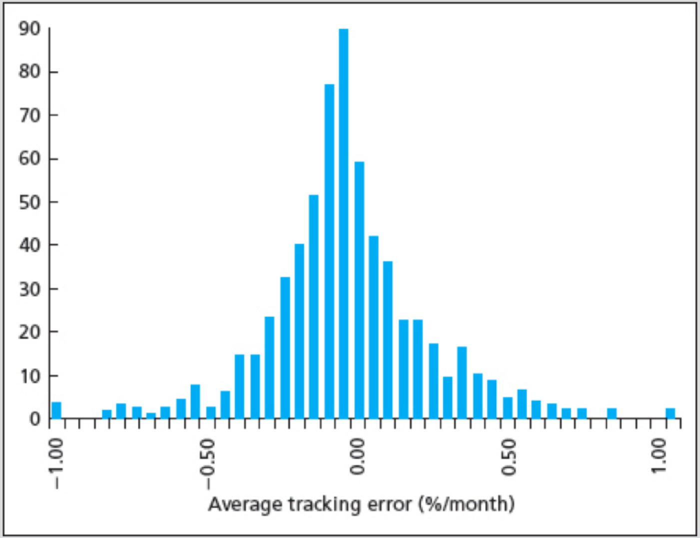
Figure 18.5 Tracking error of mutual funds

FIGURE 18.5 Average tracking error, 636 mutual funds, 1985–1989



Morningstar Risk-Adjusted Return

$$MRAR(\gamma) = \left[\frac{1}{T} \sum_{t=1}^{T} \left(\frac{1 + r_t}{1 + r_{ft}} \right)^{-\gamma} \right]^{\frac{12}{\gamma}} - 1$$

where

$$\gamma = \text{measure of risk aversion}$$

- Uses average excess monthly returns
- Can be thought of as the risk-free equivalent excess return of P with a certain risk aversion (γ)

Morningstar Risk-Adjusted Return

- Company peer groups established based on Morningstar style definitions
- Risk-adjusted performance ranked; then stars assigned according to table

Percentile	Stars
0-10	1
10-32.5	2
32.5-67.5	3
67.5-90	4
90-100	5

Performance Measurement With Changing Portfolio Composition

- We need a very long observation period to measure performance with any precision, even if the return distribution is stable with a constant mean and variance.
- What if the mean and variance are not constant? We need to keep track of portfolio changes.

Performance Measurement With Changing Portfolio Composition

- Problems with Performance Measures
  - The measures assume fund maintains constant level of risk
  - Particularly problematic for funds engaging in active asset allocation
- In large universe of funds, some will have abnormal performance each period by chance
- Survivorship bias
  - Upward bias in average fund performance due to failure to account for failed funds over sample period

Example 18.2. Risk Measurement with Changing Portfolio Composition

EXAMPLE 18.2

Risk Measurement with Changing Portfolio Composition

Suppose the Sharpe measure of the passive strategy (investing in a market-index fund) is .4. A portfolio manager is in search of a better, active strategy. Over an initial period of, say, four quarters, he executes a low-risk or defensive strategy with an annualized mean excess return of 1.5% and a standard deviation of 3.4%. This makes for a Sharpe measure of .44, which beats the passive strategy.

Over the next period of another four quarters, this manager finds that a high-risk strategy is optimal, with an annual mean excess return of 8.75% and standard deviation of 20%. Here again the Sharpe measure is .44. Over the two years, our manager maintains a better-than-passive Sharpe measure.

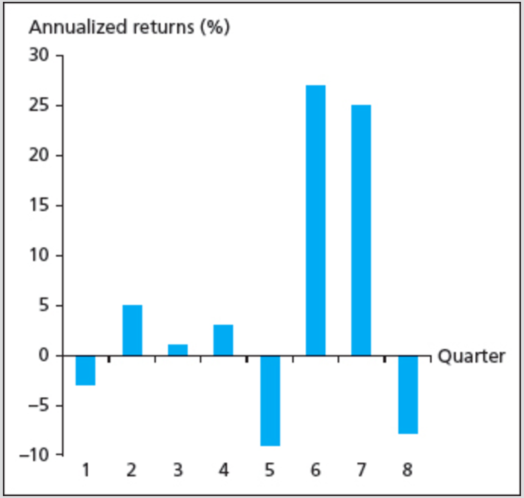
Example 18.2. Risk Measurement with Changing Portfolio Composition

Figure 18.7 shows a pattern of (annualized) quarterly returns that is consistent with our description of the manager's strategy over two years. In the first four quarters, the excess returns are – 3%, 5%, 1%, and 3%, consistent with the predicted mean and SD. In the next four quarters, the excess returns are – 9%, 27%, 25%, and – 8%, also consistent with predictions for the higher-volatility period. Thus, *each* year exhibits a Sharpe measure of .44.

But if we treat the eight-quarter sequence as a single measurement period instead of two independent periods, the portfolio's mean and standard deviation over the full period are 5.125% and 13.8% respectively, resulting in a Sharpe measure of only .37, apparently inferior to the passive strategy!

Figure 18.7

FIGURE 18.7 Portfolio returns. In the first four quarters, the firm follows a low-risk, low-return policy. In the next four quarters, it shifts to a high-risk, high-return policy.



Performance Manipulation

- A manipulation-proof performance measure (MPPM) must fulfill four requirements:
  1. The measure should produce a single-value score to rank a portfolio.
  2. The score should not depend on the dollar value of the portfolio.
  3. An uninformed investor should not expect to improve the expected score by deviating from the benchmark portfolio.
  4. The measure should be consistent with standard financial market equilibrium conditions.

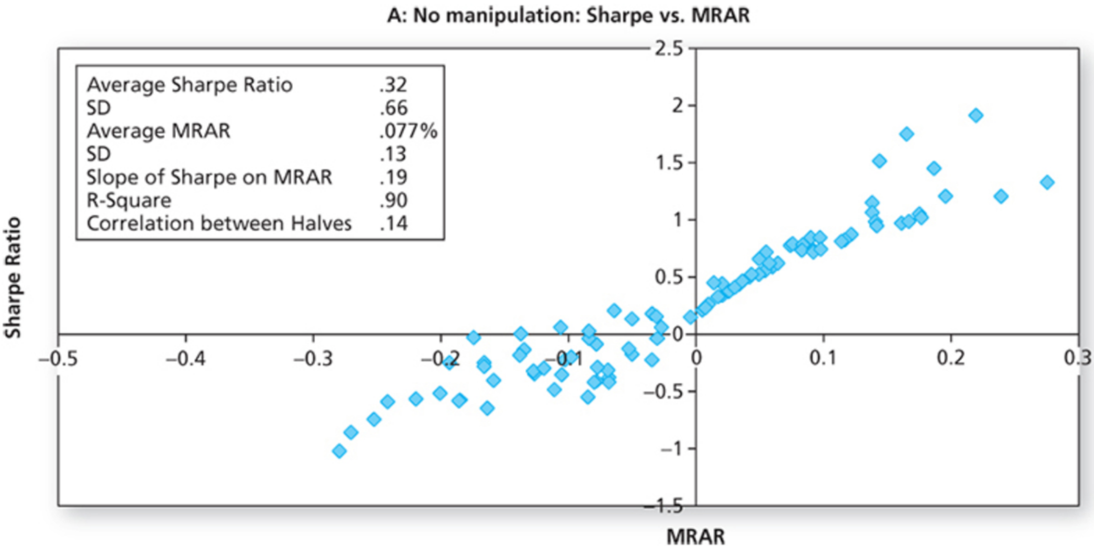
Performance Manipulation and the MRAR

- Assumption: Rates of return are independent and drawn from same distribution.
- Managers may employ strategies to improve performance at the loss of investors.
- Ingersoll, et al. show how all but one of the performance measures can be manipulated.
- Using leverage to increase potential returns.
- MRAR fulfills requirements of the MPPM

Figure Panel A Description

- Scatter of Sharpe ratios vs. MRAR of 100 portfolios based on statistical simulation.
- Thirty-six excess returns were randomly generated for each portfolio, all with an annual expected return of 7% and SDs varying from 10% to 30%.
- Thus the true Sharpe ratios of these simulated “mutual funds” are in the range of 0.23 to 0.70,
- Because of sampling variation, the actual 100 Sharpe ratios in the simulation differ quite a bit from these population parameters;
- The correlation between the measures was .94, suggesting that Sharpe ratios track MRAR quite well.

Figure Panel A No Manipulation: Sharpe vs. MRAR



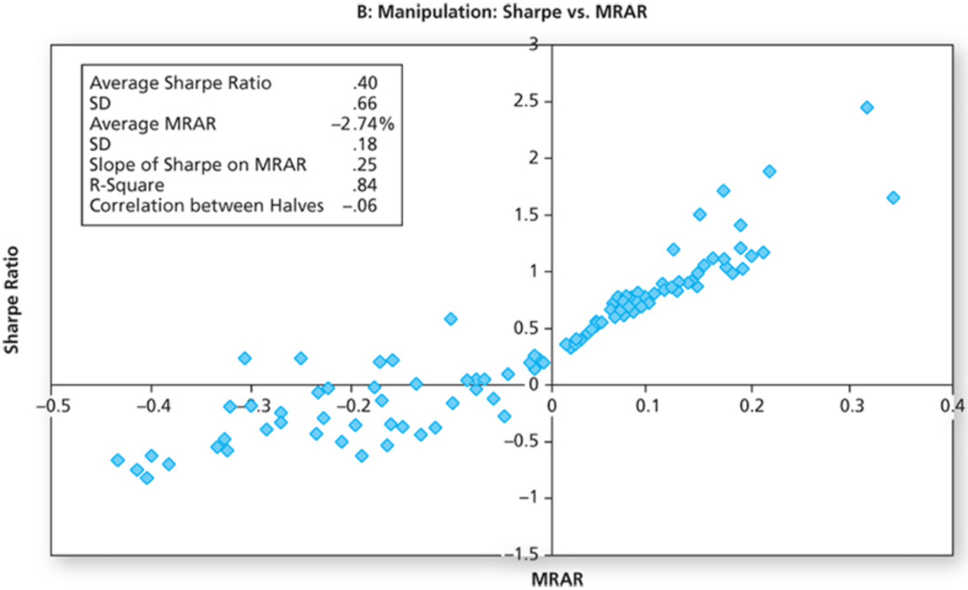
Manipulating performance

- Imagine a manager already partway into an evaluation period.
- Measures (SR,etc.) are known for the first half.
- Increasing leverage will overweigh performance in second half, both good and bad.
- Therefore, managers will wish to increase leverage in the latter part of the period if early returns are poor.
- Conversely, good first-part performance calls for deleveraging to increase the weight on the initial period.

Figure Panel B Description

- One leverage change is allowed after initial performance is observed
- For high-positive initial MRARs, the switch toward risk-free investments preserves the first-half high Sharpe ratios
- For the large-negative initial MRARs, when leverage ratios are increased, we see two effects.
  1. MRARs look worse because of cases where the high leverage backfired and worsened the MRARs
  2. Sharpe ratios look better
     - Some Sharpe ratios move from negative to positive
     - For others the increased SD in the second period reduced the absolute value of the negative Sharpe ratios

Figure Panel B Manipulation: Sharpe vs. MRAR



Performance Attribution

- A common attribution system decomposes performance into several components:
  1. Allocation choices across broad asset classes.
  2. Industry or sector choice within each market.
  3. Security choice within each sector.
  4. Timing

Attributing Performance to Components

- Set up a ‘Benchmark’ or ‘Bogey’ portfolio:
  - Select a benchmark index portfolio for each asset class.
  - Choose weights based on market expectations.
  - Choose a portfolio of securities within each class by security analysis.
  - Calculate the return on the ‘Bogey’ and on the managed portfolio.
  - Explain the difference in return based on component weights or selection.
  - Summarize the performance differences into appropriate categories.

Components of Performance Attribution

- Return on bogey portfolio (fixed weights in each asset class):

$$r_B = \sum_{i=1}^{n} w_{Bi} r_{Bi}$$

- Return on managed portfolio

$$r_P = \sum_{i=1}^{n} w_{Pi} r_{Pi}$$

- Difference in the two rates:

$$r_P - r_B = \sum_{i=1}^{n} (w_{Pi} r_{Pi} - n w_{Bi} r_{Bi}) \tag{9}$$

Table 18.4 Components of Performance Attribution

TABLE 18.4 Performance of the managed portfolio

Bogey Performance and Excess Return

Component	Benchmark Weight	Return of Index during Month (%)
Equity (S&P 500)	.60	5.81
Bonds (U.S. Aggregate Index)	.30	1.45
Cash (money market)	.10	0.48
Bogey = (.60 × 5.81) + (.30 × 1.45) + (.10 × .48) = 3.97%		
Return of managed portfolio		5.34%
–Return of bogey portfolio		3.97
Excess return of managed portfolio		1.37%

The Magellan Fund and Market Efficiency

- Fidelity's Magellan Fund outperformed the S&P 500 in eleven of the thirteen years ending in 1989.
- The fund however is not randomly selected–it is selected because it was the winner.
- Should then the benchmark be the S&P 500?
- Coin flipping analogy:
  - If fifty contestants were to flip a coin thirteen times, and the winner were to flip eleven heads out of thirteen, we would not consider that evidence that the winner's coin was biased.
- When is a manager's performance so good that it cannot be chance?

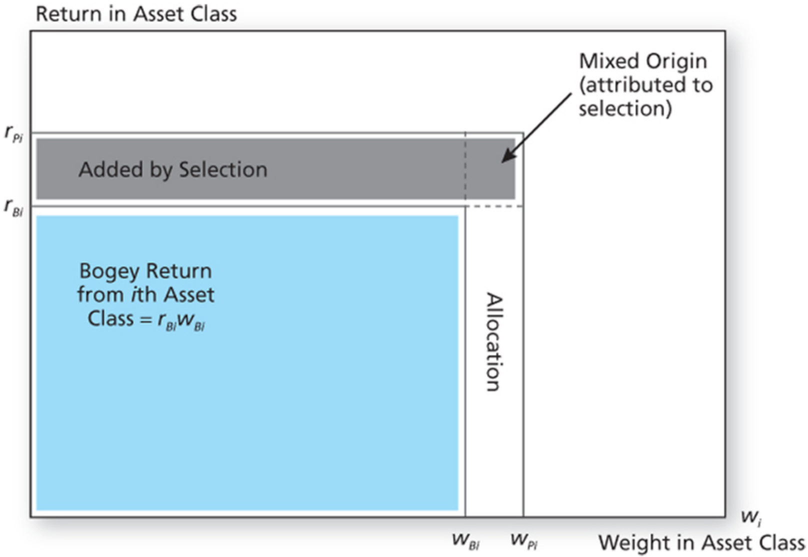
Probability Distribution of Successful Years out of 13 to be Selected as Best Manager

	Managers in Contest			
Winning Years	**50**	**100**	**250**	**500**
8	0.1%	0	0	0
9	9.2	0.9	0	0
10	47.4	31.9	5.7	0.2
11	34.8	51.3	59.7	42.3
12	7.7	14.6	31.8	51.5
13	0.8	1.2	2.8	5.9
Mean winning years of best performer	10.43	10.83	11.32	11.63

Performance Attribution

- Superior performance is achieved by:
  - overweighting assets in markets that perform well
  - underweighting assets in poorly performing markets

Performance Attribution of i^{th} Asset Class



Contribution to Returns

Contribution from asset allocation	$(w_{Pi} - w_{Bi})r_{Bi}$
+ Contribution from security selection	$w_{Pi}(r_{Pi} - r_{Bi})$
= Total contribution from asset class i	$w_{Pi}r_{Pi} - w_{Bi}r_{Bi}$

Table 18.5 Performance Attribution

TABLE 18.5 Performance attribution

A. Contribution of Asset Allocation to Performance

Market	(1) Actual Weight in Market	(2) Benchmark Weight in Market	(3) Excess Weight	(4) Index Return (%)	(5) = (3) × (4) Contribution to Performance (%)
Equity	.70	.60	.10	5.81	.5810
Fixed-income	.07	.30	−.23	1.45	−.3335
Cash	.23	.10	.13	0.48	.0624
Contribution of asset allocation					.3099

B. Contribution of Selection to Total Performance

Market	(1) Portfolio Performance (%)	(2) Index Performance (%)	(3) Excess Performance (%)	(4) Portfolio Weight	(5) = (3) × (4) Contribution (%)
Equity	7.28	5.81	1.47	.70	1.03
Fixed-income	1.89	1.45	0.44	.07	0.03
Contribution of selection within markets					1.06

Sector and Security Selection Decisions

- Good performance (a positive contribution) derives from overweighting high-performing sectors
- Good performance also derives from underweighting poorly performing sectors.

Table 18.6 Sector Selection within Equities

TABLE 18.6 Sector allocation within the equity market

	(1)	(2)	(3)	(4)	(5) = (3) × (4)
	Beginning-of-Month Weights				
Sector	Portfolio	S&P 500	Difference in Weights	Sector Return (%)	Contribution of Sector Allocation (%)
Basic materials	0.0196	0.083	−.0634	6.9	−0.437
Business services	0.0784	0.041	.0374	7.0	0.262
Capital goods	0.0187	0.078	−.0593	4.1	−0.243
Consumer cyclical	0.0847	0.125	−.0403	8.8	−0.355
Consumer noncyclical	0.4037	0.204	.1997	10.0	1.997
Credit sensitive	0.2401	0.218	.0221	5.0	0.111
Energy	0.1353	0.142	−.0067	2.6	−0.017
Technology	0.0195	0.109	−.0895	0.3	−0.027
Total	1.0000	1.000	.0000		1.290

Excel Model of Performance Attribution

	B	C	D	E	F
1		Chapter 18			
2		Performance Attribution			
3				Contribution to	
4		Weight	Return on	Portfolio	
5	Index	Benchmark	Index	Return	
6	S&P500	0.6	5.8100%	3.4860%	
7	Aggregate Index	0.3	1.4500%	0.4350%	
8	Money Market	0.1	0.4800%	0.0480%	
9					
10				3.9690%	
11					
12				Contribution to	
13		Portfolio	Actual	Portfolio	
14		Weight	Return	Return	
15		0.7	7.2800%	5.0960%	
16		0.07	1.8900%	0.1323%	
17		0.23	0.4800%	0.1104%	
18					
19				5.3387%	
20					
21				1.3697%	
22					
23					
24	Contribution of Asset Allocation				
25	Actual Weight	Benchmark	Excess	Market	Performance
26	in Portfolio	Weight	Weight	Return	Contribution
27	0.7	0.6	0.1	5.8100%	0.5810%
28	0.07	0.3	-0.23	1.4500%	-0.3335%
29	0.23	0.1	0.13	0.4800%	0.0624%
30					
31					0.3099%

http://somfin.gmu.edu/courses/fnan311/excel/ss18.xlsx

Excel Questions

1. What would happen to the contribution of asset allocation to overall performance if the actual weights had been 70/17/13 in the three markets rather than 75/12/13? Explain your result.

2. Show what would happen to the contribution of security selection to performance if the actual return on the equity portfolio had been 7.5% instead of 6.5% and the return on the S&P 500 had been 6.81% instead of 5.81%. Explain your result.

TABLE 18.7 Portfolio attribution: summary

		Contribution (basis points)
1. Asset allocation		31.0
2. Selection		
a. Equity excess return		
i. Sector allocation	129	
ii. Security selection	18	
	$147 \times .70$ (portfolio weight) =	102.9
b. Fixed-income excess return	$44 \times .07$ (portfolio weight) =	3.1
Total excess return of portfolio		137.0

CONCEPT check 18.2

a. Suppose the benchmark weights had been set at 70% equity, 25% fixed-income, and 5% cash equivalents. What then would be the contributions of the manager's asset allocation choices?

b. Suppose the S&P 500 return had been 5%. Recompute the contribution of the manager's security selection choices.

Table Potential Value of Market Timing

TABLE 18.8 Performance of cash, stocks, and perfect-timing strategies

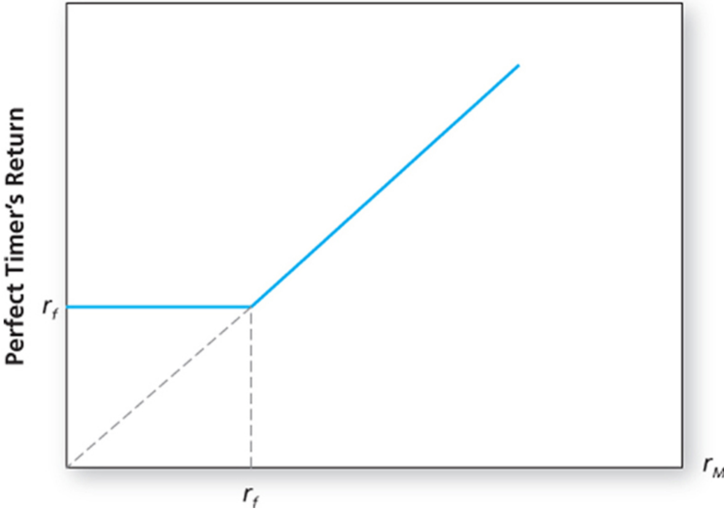
	Family/Strategy		
	A. Cash	**B. Stocks**	**C. Perfect Timing**
I. Family fund as of the end of 2008			
Final proceeds	$20	$1,626	$36,699,302,473
II. Annualized monthly rate-of-return statistics (%)			
Geometric average	3.71	9.44	34.54
Arithmetic average	3.71	11.48	35.44
Minimum monthly rate*	−0.03	−28.73	−0.03
Maximum monthly rate†	1.52	41.65	41.65
Average excess return	0.00	7.77	31.73
Standard deviation	3.54	19.38	12.44

Beginning with $1 on December 1, 1926 and ending on December 31, 2008

Figure 18.8 Rate of Return of a Perfect Market Timer

Figure 18.8 The perfect The perfect timer invests 100% in either the safe asset or the equity portfolio. The payoff is the same as if investing in T-Bills and a Call option with exercise price $X = S_0(1 + r_f)$:

	$S_T < X$	$S_T \geq X$
Bills	$S_0(1 + r_f)$	$S_0(1 + r_f)$
Call	0	$S_T - X$
Total	$S_0(1 + r_f)$	S_T



Valuing Perfect Market Timing Ability with Options

- We can use option-pricing to assign a dollar value to perfect timing ability.
- Using continuous compounding, the exercise price is $\$1e^{rT}$, i.e. the T-Bill return on \$1.
- Using the Black-Scholes formula:

$$C = S_0 N(d_1) - Xe^{-rt} N(d_2)$$

$$\text{where} \quad d_1 = \frac{ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\text{MV(Perfect timer per \$ of assets)} = C = 2N({}^1\!/\!{}_2\sigma_M\sqrt{T}) - 1 \tag{6}$$

Valuing Imperfect Market Timing Ability with Options

- Option-pricing also enables us to assign value to less-than-perfect timers.
- Let P_1 be the proportion of the correct forecasts of bull markets and P_2 the proportion for bear markets. Then:

$$\begin{aligned} \text{MV(imperfect timer)} &= (P_1 + P_2 - 1) \times C \\ &= (P_1 + P_2 - 1) \times \left[2N(^1/_2 \sigma_M \sqrt{T}) - 1 \right] \end{aligned} \tag{7}$$

- If the timer does not shift fully from one asset to the other, but shifts only a fraction ω between T-bills and equities:

$$\text{MV(imperfect timer)} = \omega(P_1 + P_2 - 1) \times \left[2N(^1/_2 \sigma_M \sqrt{T}) - 1 \right]$$

Market Timing Test, Treynor and Mazuy

- In its pure form, market timing involves shifting funds between a market-index portfolio and a safe asset. Treynor and Mazuy:

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)^2 + e_P$$

Henriksson and Merton:

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)D + e_P$$

where D is a dummy variable that equals 1 for $r_M > r_f$ and zero otherwise.

Figure Market Timing and Characteristic Lines

Panel A: No market timing, beta is constant

Panel B: Market timing, beta increases with market excess return

Panel C: Market timing, with only two values of beta