Chapter 9, The Capital Asset Pricing Model

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Chapter Overview

The CAPM

- Assumptions
- Equilibrium Conditions
- The Capital Market Line
- Risk Premiums
- The CAPM Equation
- The Security Market Line
- CAPM and Single-Index Model

Extensions

CAPM and the Academia

CAPM and the Industry

The Capital Asset Pricing Model (CAPM)

- It is the equilibrium model that underlies all modern financial theory
- Derived using principles of diversification with simplified assumptions
- Markowitz, Sharpe, Lintner and Mossin are researchers credited with its development

Assumptions

- Investors optimize portfolios a la Markowitz
- Investors use identical input list for efficient frontier
- Same risk-free rate, tangent CAL and risky portfolio
- Market portfolio is aggregation of all risky portfolios and has same weights

Table 9.1. Assumptions of the CAPM

Table 9.1

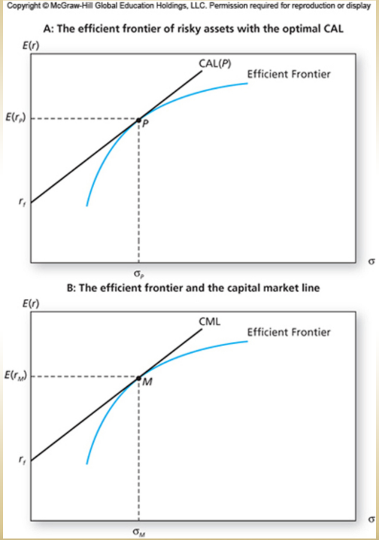
The assumptions of the CAPM

1. Individual behavior
   a. Investors are rational, mean-variance optimizers.
   b. Their planning horizon is a single period.
   c. Investors have homogeneous expectations (identical input lists).
2. Market structure
   a. All assets are publicly held and trade on public exchanges, short positions are allowed, and investors can borrow or lend at a common risk-free rate.
   b. All information is publicly available.
   c. No taxes.
   d. No transaction costs.

Resulting Equilibrium Conditions

- All investors will hold the same portfolio for risky assets – market portfolio
- Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value

Figure 9.1 The Efficient Frontier and the CML



Market Risk Premium

- The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the investor:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} \tag{9.1}$$

$$E(R_M) = \bar{A}\sigma_M^2 \tag{9.2}$$

where

$E(R_M) = E(r_M) - r_f$ is the risk premium (expected excess return) on the market

σ_M^2 is the variance of the market portfolio

$\bar{A}$ is the average degree of risk aversion across investors

Return and Risk for Individual Securities

- The risk premium on individual securities is a function of the individual security's contribution to the risk of the market portfolio.
- An individual security's risk premium is a function of the covariance of returns with the assets that make up the market portfolio.

GE Example

- Contribution of GE to market variance

Portfolio Weights	w_1	w_2	...	w_{GE}	...	w_n
w_1	$Cov(R_1, R_1)$	$Cov(R_1, R_2)$	...	$Cov(R_1, R_{GE})$	...	$Cov(R_1, R_n)$
w_2	$Cov(R_2, R_1)$	$Cov(R_2, R_2)$	...	$Cov(R_2, R_{GE})$	...	$Cov(R_2, R_n)$
⋮	⋮	⋮		⋮		⋮
w_{GE}	$Cov(R_{GE}, R_1)$	$Cov(R_{GE}, R_2)$	...	$Cov(R_{GE}, R_{GE})$	...	$Cov(R_{GE}, R_n)$
⋮	⋮	⋮		⋮		⋮
w_n	$Cov(R_n, R_1)$	$Cov(R_n, R_2)$	...	$Cov(R_n, R_{GE})$	...	$Cov(R_n, R_n)$

GE Example

- Covariance of GE return with the market portfolio:

$$\sum_{i=1}^{n} w_i Cov(R_i, R_{GE}) = \sum_{i=1}^{n} Cov(w_i R_i, R_{GE}) = Cov\left(\underbrace{\sum_{i=1}^{n} w_i R_i}_{R_M}, R_{GE}\right) \tag{9.4}$$

- Therefore, the reward-to-risk ratio for investments in GE would be:

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} Cov(R_{GE}, R_M)} = \frac{E(R_{GE})}{Cov(R_{GE}, R_M)}$$

GE Example

- Reward-to-risk ratio for investment in market portfolio:

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(R_M)}{\sigma_M^2} \tag{9.5}$$

- Reward-to-risk ratios of GE and the market portfolio should be equal:

$$\frac{E(R_{GE})}{Cov(R_{GE}, R_M)} = \frac{E(R_M)}{\sigma_M^2} \tag{9.6}$$

rearrange

$$E(R_{GE}) = \frac{Cov(R_{GE}, R_M)}{\sigma_M^2} E(R_M) \tag{9.7}$$

GE Example, The CAPM Equation

$$E(r_{GE}) = r_f + \beta_{GE}\left[E(r_M) - r_f\right] \tag{9.8}$$

Expected Return-Beta Relationship

- CAPM holds for the overall portfolio because:

$$
\begin{aligned}
w_1 E(r_1) &= w_1 r_f + w_1 \beta_1 [E(r_M) - r_f] \\
+w_2 E(r_2) &= w_2 r_f + w_2 \beta_2 [E(r_M) - r_f] \\
&\vdots \\
+w_n E(r_n) &= w_n r_f + w_n \beta_n [E(r_M) - r_f] \\
\hline
\end{aligned}
$$

$$E(r_p) = r_f + \beta_p [E(r_M) - r_f]$$

Figure 9.2 The Security Market Line

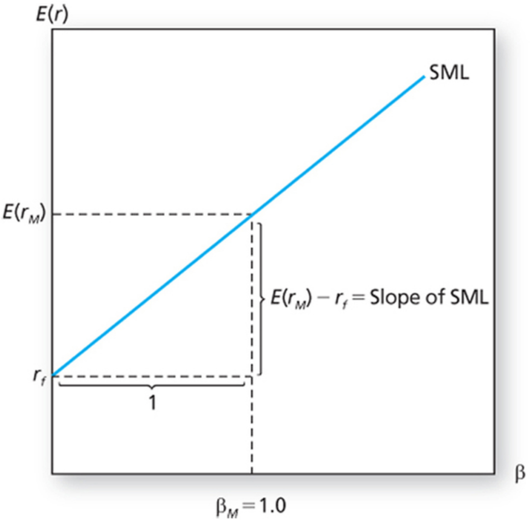
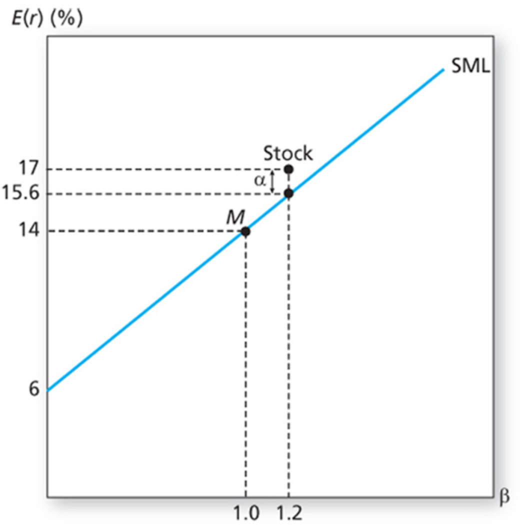


Figure 9.3 The SML and a Positive-Alpha Stock



CAPM and Single-Index Model

- To move from expected to realized returns, use the index model in excess return form:

$$R_i = \alpha_i + \beta_i R_M + e_i \tag{9.9}$$

- The index model beta coefficient is the same as the beta of the CAPM expected return-beta relationship.

Zero-Beta CAPM

- Every efficient portfolio has a "companion" portfolio on the bottom (inefficient) half of the frontier, with which it is uncorrelated

$$E(r_i) - E(r_Z) = [E(R_M) - E(R_Z)]\frac{Cov(r_i, r_M)}{\sigma_M^2} = \beta_i[E(R_M) - E(R_Z)] \quad (9.12)$$

where Z is the market's 'zero-beta' portfolio

- Helps to explain positive alphas on low beta stocks and negative alphas on high beta stocks
- Consideration of labor income and non-traded assets

Labor Income and Non-Traded Assets

- If private assets are similar to traded ones, the CAPM holds
- If not, then a traded portfolio that best hedges against private asset risk will be in excess demand (high price, low return, negative alpha)
- Uncertainty of labor income more difficult to hedge—stocks of labor-intensive industries may serve as hedge and in high demand
- SML with labor income (Meyers, 1972):

$$E(R_i) = E(R_M)\frac{Cov(R_i, R_M) + \frac{P_H}{P_M}Cov(R_i, R_H)}{\sigma_M^2 + \frac{P_H}{P_M}Cov(R_M, R_H)} \tag{9.13}$$

where P_H is the value of human capital, P_M, the value of the market portfolio, and R_H, the excess return on aggregate human capital

Multiperiod Model and Hedge Portfolios

- Merton (1973) relaxed the single-period assumption – individuals optimize lifetime consumption – intertemporal CAPM (ICAPM)
- Additional sources of risk:
  - Changes in investment opportunities – future risk-free rates, market risk – which may affect future spending plans
  - Prices of consumption goods (inflation risk)
- Portfolios which co-vary with (track) these risks act as additional factors:

$$E(R_i) = \beta_{iM} E(R_M) + \sum_{k=1}^{K} \beta_{ik} E(R_k) \tag{9.14}$$

Consumption-based CAPM

- Center the model directly on consumption – allocate current wealth between today's consumption and the savings and investment that will support future consumption.
- Write the risk premium on an asset as a function of its "consumption risk"

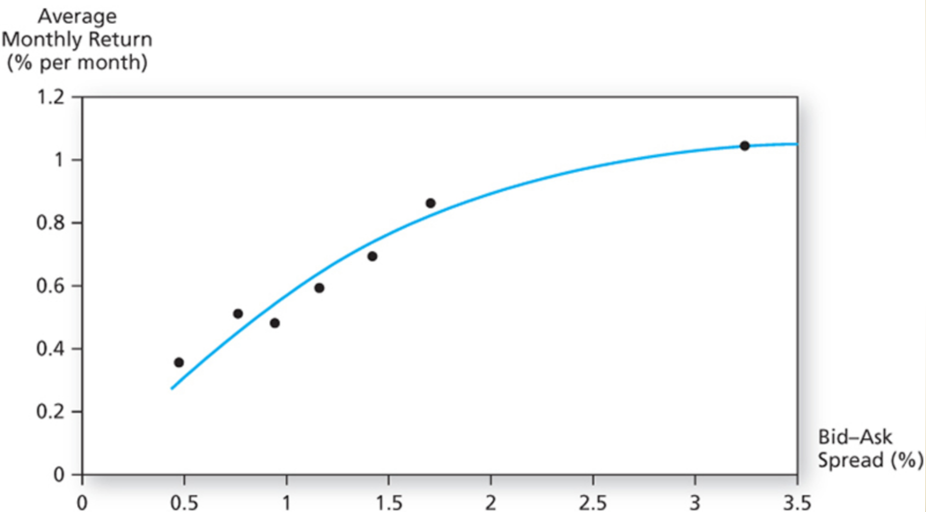
$$E(R_i) = \beta_{iC} RP_C \tag{9.15}$$

- Theoretically, the CCAPM answers all asset pricing questions. Empirically, performs very poorly

Liquidity and the CAPM

- Liquidity: The ease and speed with which an asset can be sold at fair market value
- Illiquidity Premium: Discount from fair market value the seller must accept to obtain a quick sale.
  - Measured partly by bid-asked spread
  - As trading costs are higher, the illiquidity discount will be greater.

Figure 9.4 Illiquidity and Average Returns



Liquidity Risk

- In a financial crisis, liquidity can unexpectedly dry up.
- When liquidity in one stock decreases, it tends to decrease in other stocks at the same time.
- Investors demand compensation for liquidity risk
    - Liquidity betas

The CAPM and the Academic World

- The theoretical market portfolio impossible to observe (Roll 1977)
- Betas are unobservable, likely time varying
- Fama and French (1993) three factor model
- Generally the unconditional CAPM fails, some support for the conditional CAPM

The CAPM and the Investment Industry

- Relies on the single-index CAPM model
- Most investors don't beat the index portfolio. Hence it can be considered efficient and used:
  - for diversification
  - as a benchmark
  - to assess fair compensation for risk
  - for price regulation

Figure 9.5 Estimates of Mutual Fund Alphas