Chapter 24, Portfolio Performance Evaluation

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Chapter Overview

Introduction

- If markets are efficient, investors must be able to measure asset management performance
- Two common ways to measure average portfolio return:
  1. Time-weighted returns
  2. Dollar-weighted returns
- Returns must be adjusted for risk.

Dollar- and Time-Weighted Average Returns

- Time-weighted returns
  - The geometric average is a time-weighted average.
  - Each period's return has equal weight.

$$(1 + r_G)^n = (1 + r_1)(1 + r_2)\ldots(1 + r_n)$$

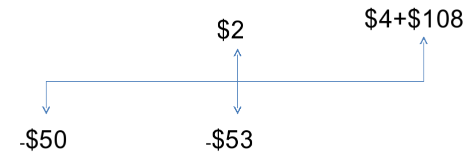
Dollar- and Time-Weighted Average Returns

- Dollar-weighted returns
  - Internal rate of return considering the cash flow from or to investment
  - Returns are weighted by the amount invested in each period:

$$PV = \frac{C_1}{(1+r)^1} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

Example of Multiperiod Returns

Time	Outlay
0	\$50 to purchase first share
1	\$53 to purchase second share a year later
	Proceeds
1	\$2 dividend from initially purchased share
2	\$4 dividend from the 2 shares held in the second year, plus \$108 received from selling both shares at \$54 each



- Dollar-weighted return:

$$50 + \frac{53}{1+r} = \frac{2}{1+r} + \frac{112}{(1+r)^2} \qquad r = 7.117\%$$

Time-Weighted Return

$$r_1 = \frac{53 - 50 + 2}{50} = 10\%$$
$$r_2 = \frac{54 - 53 + 2}{53} = 5.66\%$$
$$r_G = [(1.1)(1.0566)]^{1/2} - 1 = 7.81\%$$

- The dollar-weighted average is less than the time-weighted average in this example because more money is invested in year two, when the return was lower.

Time-Weighted Return

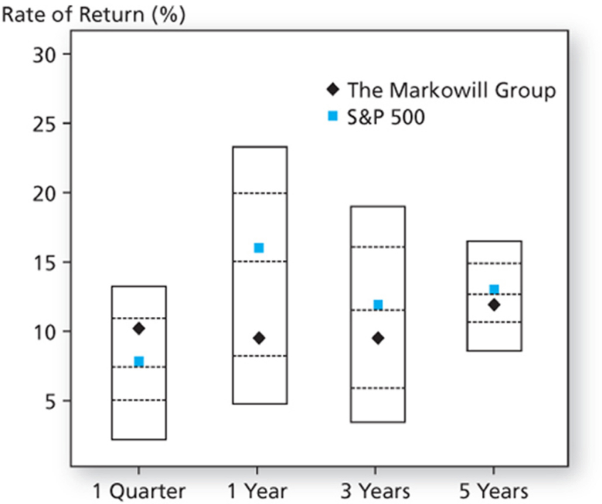
- Households should maintain a spreadsheet of time-dated cash flows (in and out) to determine the effective rate of return for any given period.
- Examples include:
  - IRA, 401(k), 529

Adjusting Returns for Risk

- The simplest and most popular way to adjust returns for risk is to compare the portfolio's return with the returns on a comparison universe.
- The comparison universe is a benchmark composed of a group of funds or portfolios with similar risk characteristics, such as growth stock funds or high-yield bond funds.

Figure 24.1 Universe Comparison

Figure 24.1. The chart summarizes performance rankings over four periods: 1 quarter, 1 year, 3 years, and 5 years. The top and bottom lines of each box are drawn at the rate of return of the 95th and 5th percentile managers. The three dashed lines correspond to the rates of return of the 75th, 50th (median), and 25th percentile managers.



Risk Adjusted Performance: Sharpe

$$S = \frac{\bar{r}_P - \bar{r}_f}{\bar{\sigma}_P}$$

where

$$\bar{r}_P = \text{Average return on the portfolio}$$
$$\bar{r}_f = \text{Average risk-free rate}$$
$$\bar{\sigma}_P = \text{standard deviation of portfolio return}$$

Risk Adjusted Performance: Treynor

$$\text{Treynor Measure} = \frac{\bar{r}_P - \bar{r}_f}{\beta_P}$$

where

$$\bar{r}_P = \text{Average return on the portfolio}$$
$$\bar{r}_f = \text{Average risk-free rate}$$
$$\beta_P = \text{weighted average beta for portfolio}$$

Risk Adjusted Performance: Jensen

Jensen's Alpha

$$\alpha_P = \bar{r}_P - [\bar{r}_f + \beta_P(\bar{r}_M - \bar{r}_f)]$$

where

$$\begin{aligned}
\alpha_P &= \text{Alpha for the portfolio} \\
\bar{r}_P &= \text{Average return on the portfolio} \\
\bar{r}_f &= \text{Average risk-free rate} \\
\beta_P &= \text{weighted average beta for portfolio} \\
\bar{r}_M &= \text{Average return on the market index portfolio}
\end{aligned}$$

Information Ratio

$$\text{information Ratio} = \frac{\alpha_P}{\sigma(e_P)}$$

where

$$\alpha_P = \text{Alpha for the portfolio}$$
$$\bar{\sigma}_P = \text{standard deviation of unexpected portfolio return}$$

- The information ratio divides portfolio alpha by its nonsystematic risk.
- Nonsystematic risk could, in theory, be eliminated by diversification.

Morningstar Risk-Adjusted Return

$$MRAR(\gamma) = \left[\frac{1}{T} \sum_{t=1}^{T} \left(\frac{1 + r_t}{1 + r_{ft}} \right)^{-\gamma} \right]^{\frac{12}{\gamma}} - 1$$

where

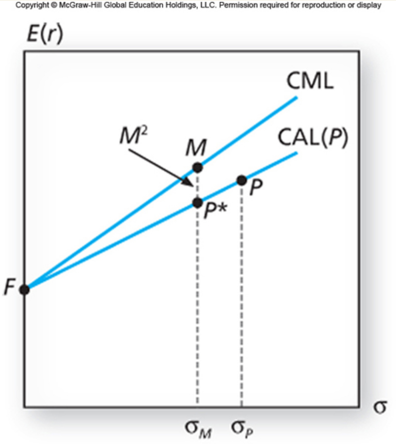
$\gamma =$ measure of risk aversion

- Uses average excess monthly returns
- Can be thought of as the risk-free equivalent excess return of P with a certain risk aversion (γ)

The M^2 Measure

$$M_P^2 = r_{P*} - r_M \qquad \sigma_{P*} = \sigma_M \tag{24.1}$$

Figure 24.2. The M^2 measure.We move down the capital allocation line corresponding to portfolio P (by mixing P with T-bills) until we reduce the standard deviation of the adjusted portfolio to match that of the market index.



Which Measure is Appropriate?

- It depends on investment assumptions
  1. If P is not diversified, then use the Sharpe measure as it measures reward to risk.
  2. If the P is diversified, non-systematic risk is negligible and the appropriate metric is Treynor’s, measuring excess return to beta.

Table 24.1 Portfolios *P* and *Q*

Table 24.1

Portfolio performance

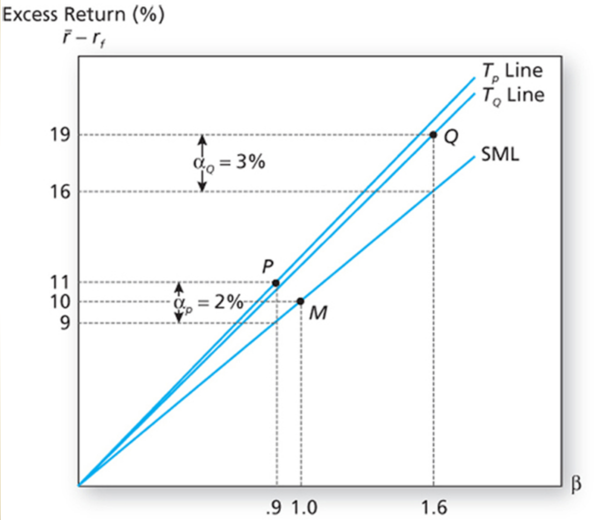
	Portfolio *P*	Portfolio *Q*	Market
Beta	.90	1.60	1.0
Excess return ($\bar{r} - \bar{r}_f$)	11%	19%	10%
Alpha*	2%	3%	0

*Alpha = Excess return − (Beta × Market excess return)
$= (\bar{r} - \bar{r}_f) - \beta(\bar{r}_M - \bar{r}_f) = \bar{r} - [\bar{r}_f + \beta(\bar{r}_M - \bar{r}_f)]$

- Portfolio *Q* has higher alpha.
- We focus on beta because *P* and *Q* are two of many fund portfolios.

Figure 24.3 Treynor's measure

Figure 24.3 shows the *T*-lines (combinations of portfolios with T-bills) for portfolios *P* and *Q*. *P* has a steeper *T*-line; despite its lower alpha, *P* is a better portfolio after all. For any given beta, a mixture of *P* with T-bills will give a better alpha than a mixture of *Q* with T-bills.



Treynor vs. Sharpe

	Treynor (T_p)	Sharpe* (S_p)
Relation to alpha	$\dfrac{E(r_p) - r_f}{\beta_p} = \dfrac{\alpha_p}{\beta_p} + T_M$	$\dfrac{E(r_p) - r_f}{\sigma_p} = \dfrac{\alpha_p}{\sigma_p} + \rho S_M$
Deviation from market performance	$T_p^2 = T_p - T_M = \dfrac{\alpha_p}{\beta_p}$	$S_p - S_M = \dfrac{\alpha_p}{\sigma_p} - (1 - \rho)S_M$

*ρ denotes the correlation coefficient between portfolio *P* and the market, and is less than 1.

Table 24.2 Excess Returns

Table 24.2

Excess returns for portfolios *P* and *Q* and the benchmark *M* over 12 months

Month	Jane's Portfolio *P*	Alternative *Q*	Benchmark *M*
1	3.58%	2.81%	2.20%
2	−4.91	−1.15	−8.41
3	6.51	2.53	3.27
4	11.13	37.09	14.41
5	8.78	12.88	7.71
6	9.38	39.08	14.36
7	−3.66	−8.84	−6.15
8	5.56	0.83	2.74
9	−7.72	0.85	−15.27
10	7.76	12.09	6.49
11	−4.01	−5.68	−3.13
12	0.78	−1.77	1.41
Average	2.77	7.56	1.64
Standard deviation	6.45	15.55	8.84

Table 24.3 Performance Statistics

Table 24.3

Performance statistics

	Portfolio *P*	Portfolio *Q*	Portfolio *M*
Sharpe ratio	0.43	0.49	0.19
M^2	2.16	2.66	0.00
Morningstar RAR	0.30	0.80	0.07
SCL regression statistics			
Alpha	1.63	5.26	0.00
Beta	0.70	1.40	1.00
Treynor	3.97	5.38	1.64
T^2	2.34	3.74	0.00
$\sigma(e)$	2.02	9.81	0.00
Information ratio	0.81	0.54	0.00
R-SQR	0.91	0.64	1.00

Interpretation of Table 24.3

- If P or Q represents the entire investment, Q is better because of its higher Sharpe measure and better M^2.
- If P and Q are competing for a role as one of a number of subportfolios, Q also dominates because its Treynor measure is higher.
- If we seek an active portfolio to mix with an index portfolio, P is better due to its higher information ratio.

Performance Manipulation and the MRAR

- Assumption: Rates of return are independent and drawn from same distribution.
- Managers may employ strategies to improve performance at the loss of investors.
- Ingersoll, et al. show how all but one of the performance measures can be manipulated.
- Using leverage to increase potential returns.
- MRAR fulfills requirements of the MPPM

Figure 24.4 Panel A Description

- Scatter of Sharpe ratios vs. MRAR of 100 portfolios based on statistical simulation.
- Thirty-six excess returns were randomly generated for each portfolio, all with an annual expected return of 7% and SDs varying from 10% to 30%.
- Thus the true Sharpe ratios of these simulated “mutual funds” are in the range of 0.23 to 0.70,
- Because of sampling variation, the actual 100 Sharpe ratios in the simulation differ quite a bit from these population parameters;
- The correlation between the measures was .94, suggesting that Sharpe ratios track MRAR quite well.

Figure 24.4a No Manipulation: Sharpe vs. MRAR

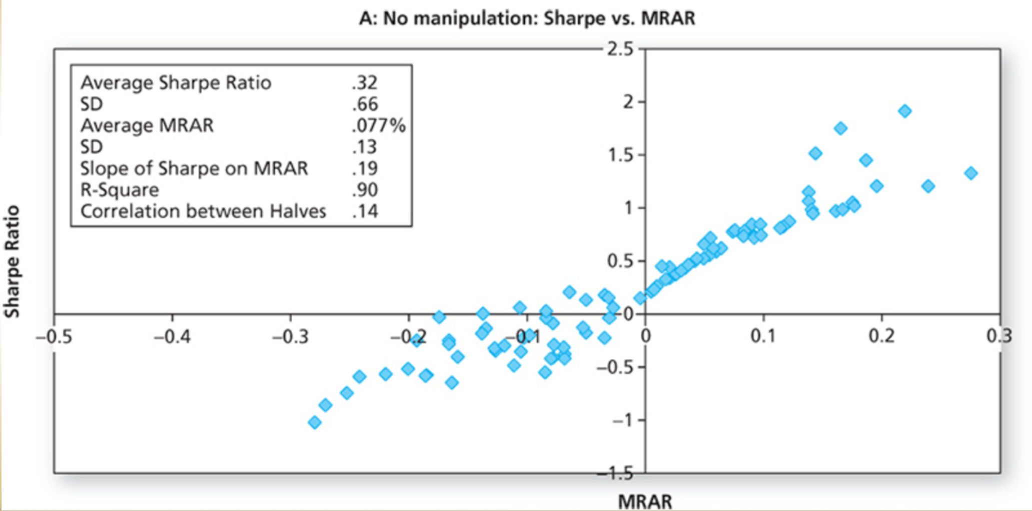


Figure 24.4b Manipulation: Sharpe vs. MRAR

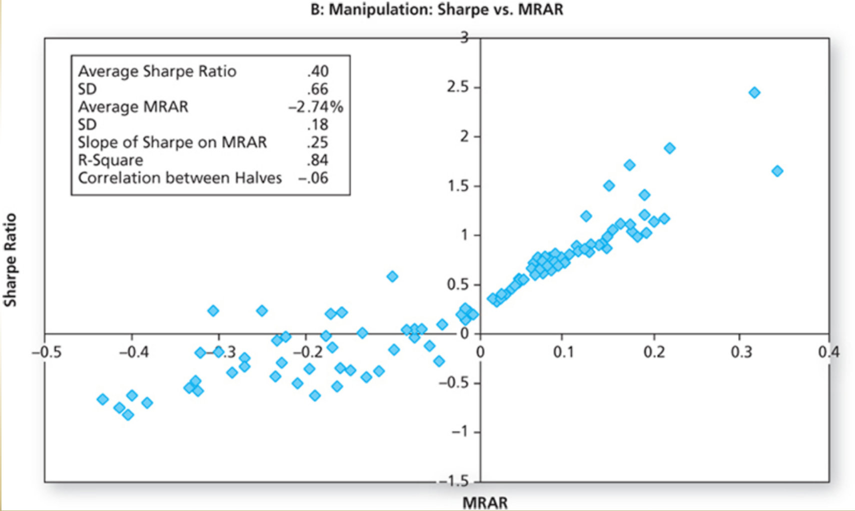


Figure 24.4 Panel B Description

- One leverage change is allowed after initial performance is observed
- For high-positive initial MRARs, the switch toward risk-free investments preserves the first-half high Sharpe ratios
- For the large-negative initial MRARs, when leverage ratios are increased, we see two effects.
  1. MRARs look worse because of cases where the high leverage backfired and worsened the MRARs
  2. Sharpe ratios look better
     - Some Sharpe ratios move from negative to positive
     - For others the increased SD in the second period reduced the absolute value of the negative Sharpe ratios

Realized Returns versus Expected Returns

- Manager's original expectations are unknown
- One observes performance only after the fact
- Need 'significance level' of performance measure–long sample period
- Average tenure of managers only 4.5 years
- Survival bias

Performance Measurement for Hedge Funds

- Let H be the active portfolio established by the hedge fund, and M the investor's baseline portfolio
- Optimal position of H in the overal hedge fund portfolio P^* is:

$$w_H = \frac{w_H^0}{1 + (1 - \beta_H) w_H^0} \qquad w_H^0 = \frac{\frac{\alpha_H}{\sigma^2(e_H)}}{\frac{E(R_M)}{\sigma_M^2}} \tag{24.2}$$

- When the hedge fund is optimally combined with the baseline portfolio, the improvement in the Sharpe measure will be determined by its information ratio:

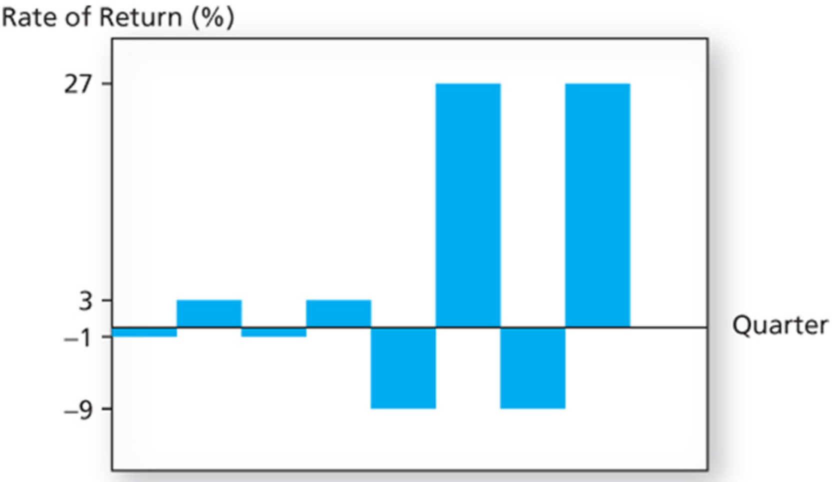
$$S_{P*}^2 = S_M^2 + \left[\frac{\alpha_H}{\sigma(e_H)} \right]^2 \tag{24.3}$$

Performance Measurement With Changing Portfolio Composition

- We need a very long observation period to measure performance with any precision, even if the return distribution is stable with a constant mean and variance.
- What if the mean and variance are not constant? We need to keep track of portfolio changes.

Figure 24.5 Portfolio Returns

Figure 24.5 The manager switches from a low-risk (SR=0.5) to a high-risk strategy (SR=0.5). However over the 8 quarters the SR is 0.37, incorrectly implying inferior performance.



Market Timing

- In its pure form, market timing involves shifting funds between a market-index portfolio and a safe asset. Treynor and Mazuy:

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)^2 + e_P$$

Henriksson and Merton:

$$r_P - r_f = a + b(r_M - r_f) + c(r_M - r_f)D + e_P$$

where D is a dummy variable that equals 1 for $r_M > r_f$ and zero otherwise.

Figure 24.6 Market Timing and Characteristic Lines

Panel A: No market timing, beta is constant

Panel B: Market timing, beta increases with market excess return

Panel C: Market timing, with only two values of beta

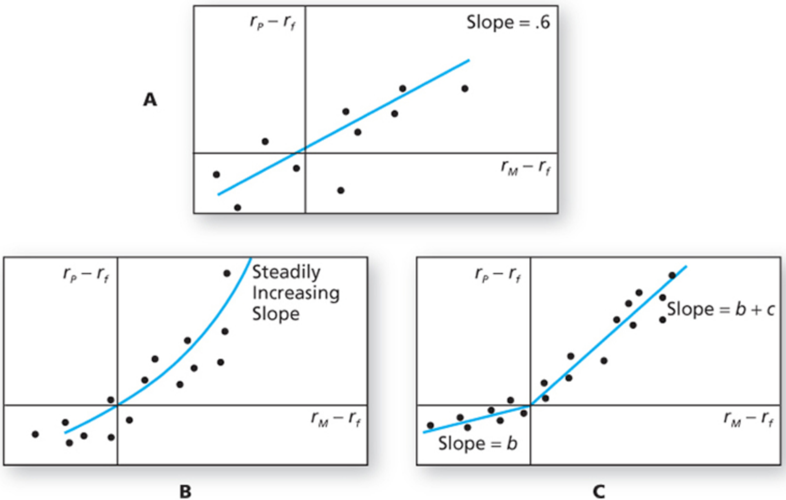


Table 24.4 Potential Value of Market Timing

Strategy	Bills	Equities	Perfect Timer	Imperfect Timer*
Terminal Value	20	2,652	352,796	8,859
Arithmetic Average	3.59	11.63	16.75	11.98
Standard Deviation	3.12	20.39	13.49	14.36
Geometric Average	3.54	9.60	16.01	11.09
LPSD (relative to bills)	0	21.18	0	17.15
Minimum†	−0.04	−44.00	−0.02	−27.09
Maximum	14.72	57.42	57.42	57.42
Skew	0.99	−0.42	0.72	0.71
Kurtosis	0.98	0.02	−0.13	1.50

Table 24.4

Performance of bills, equities, and (annual) timers—perfect and imperfect

*The imperfect timer has $P_1 = .7$, and $P_2 = .7$. $P_1 + P_2 - 1 = .4$.

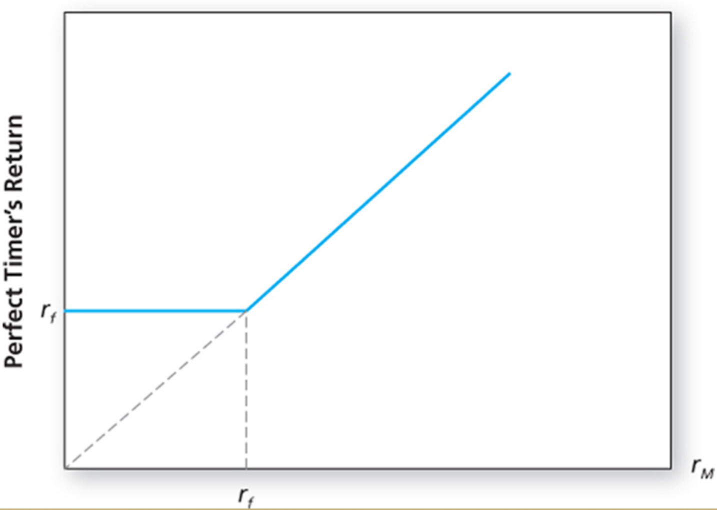
†A negative rate on "bills" was observed in 1940. The Treasury security used in the data series in these early years was actually not a T-bill but a T-bond with 30 days to maturity.

Beginning with $1 on January 1, 1927 and ending on December 31, 2012

Figure 24.7 Rate of Return of a Perfect Market Timer

Figure 24.7 The perfect The perfect timer invests 100% in either the safe asset or the equity portfolio. The payoff is the same as if investing in T-Bills and a Call option with exersize price $X = S_0(1 + r_f)$:

	$S_T < X$	$S_T \geq X$
Bills	$S_0(1 + r_f)$	$S_0(1 + r_f)$
Call	0	$S_T - X$
Total	$S_0(1 + r_f)$	S_T



Valuing Perfect Market Timing Ability with Options

- We can use option-pricing to assign a dollar value to perfect timing ability.
- Using continuous compounding, the exersize price is $\$1e^{rT}$, i.e. the T-Bill return on $1.
- Using the Black-Scholes formula:

$$C = S_0 N(d_1) - Xe^{-rt} N(d_2)$$

$$\text{where} \quad d_1 = \frac{ln(S_0/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$\text{MV(Perfect timer per \$ of assets)} = C = 2N(^1/_2\sigma_M\sqrt{T}) - 1 \qquad (24.6)$$

Valuing Imperfect Market Timing Ability with Options

- Option-pricing also enables us to assign value to less-than-perfect timers.
- Let P_1 be the proportion of the correct forecasts of bull markets and P_2 the proportion for bear markets. Then:

$$\begin{aligned} \text{MV(imperfect timer)} &= (P_1 + P_2 - 1) \times C \\ &= (P_1 + P_2 - 1) \times \left[2N({}^1\!/_2\sigma_M\sqrt{T}) - 1\right] \end{aligned} \qquad (24.7)$$

- If the timer does not shift fully from one asset to the other, but shifts only a fraction ω between T-bills and equities:

$$\text{MV(imperfect timer)} = \omega(P_1 + P_2 - 1) \times \left[2N({}^1\!/_2\sigma_M\sqrt{T}) - 1\right]$$

Style Analysis

- Introduced by William Sharpe
- Regress fund returns on indexes representing a range of asset classes.
- The regression coefficient on each index measures the fund's implicit allocation to that "style."
- Constraints on the regression coefficients: to be positive and to sum to 1.0.

Style Analysis

- R-square measures return variability due to style or asset allocation.
- The remainder is due either to security selection or to market timing.
- Intercept can be nonzero due to superior risk-adjusted abnormal return.
- Style analysis provides an alternative to performance evaluation based on the security market line (SML) of the CAPM.
- Style analysis reveals the fund’s strategy.

Table 24.5 Style Analysis for Fidelity's Magellan Fund

Style Portfolio	Regression Coefficient
T-Bill	0
Small Cap	0
Medium Cap	35
Large Cap	61
High P/E (growth)	5
Medium P/E	0
Low P/E (value)	0
Total	100
R-square	97.5

Source: Authors' calculations. Return data for Magellan obtained from **finance.yahoo.com/funds** and return data for style portfolios obtained from the Web page of Professor Kenneth French: **mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**.

Table 24.5

Style analysis for Fidelity's Magellan Fund

Figure 24.8 Fidelity Magellan Fund Cumulative Return Difference

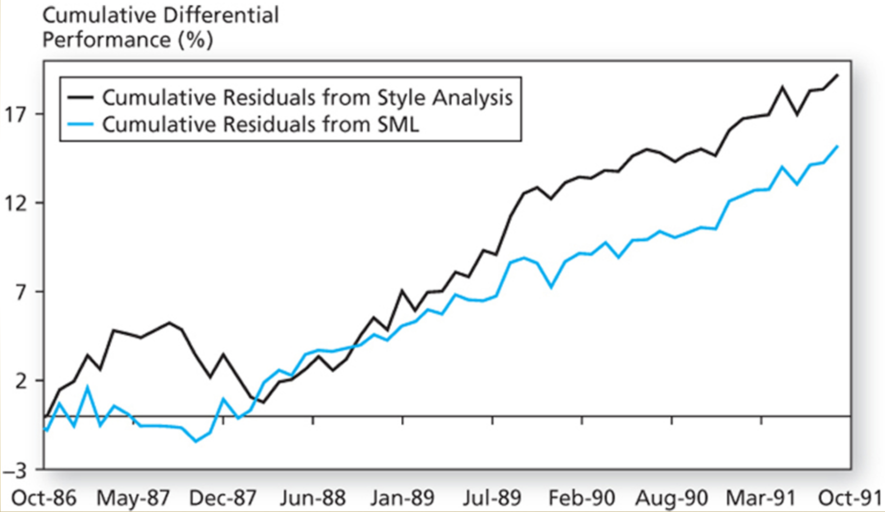
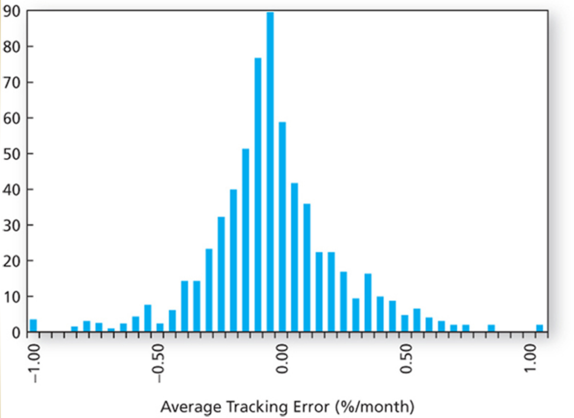


Figure 24.9 Average Tracking Error (%/month)



Tracking error (Sharpe 1992):

$$e_i = r_i - [b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{in}F_n]$$

Performance Attribution

- A common attribution system decomposes performance into three components:
  1. Allocation choices across broad asset classes.
  2. Industry or sector choice within each market.
  3. Security choice within each sector.

Attributing Performance to Components

- Set up a 'Benchmark' or 'Bogey' portfolio:
  - Select a benchmark index portfolio for each asset class.
  - Choose weights based on market expectations.
  - Choose a portfolio of securities within each class by security analysis.
  - Calculate the return on the 'Bogey' and on the managed portfolio.
  - Explain the difference in return based on component weights or selection.
  - Summarize the performance differences into appropriate categories.

Components of Performance Attribution

- Return on bogey portfolio (fixed weights in each asset class):

$$r_B = \sum_{i=1}^{n} w_{Bi} r_{Bi}$$

- Return on managed portfolio

$$r_P = \sum_{i=1}^{n} w_{Pi} r_{Pi}$$

- Difference in the two rates:

$$r_P - r_B = \sum_{i=1}^{n} (w_{Pi} r_{Pi} - w_{Bi} r_{Bi}) \tag{24.9}$$

Contribution to Returns

Contribution from asset allocation	$(w_{Pi} - w_{Bi})r_{Bi}$
+ Contribution from security selection	$w_{Pi}(r_{Pi} - r_{Bi})$
=Total contribution from asset class *i*	$w_{Pi}r_{Pi} - w_{Bi}r_{Bi}$

Figure 24.10 Performance Attribution of i^{th} Asset Class

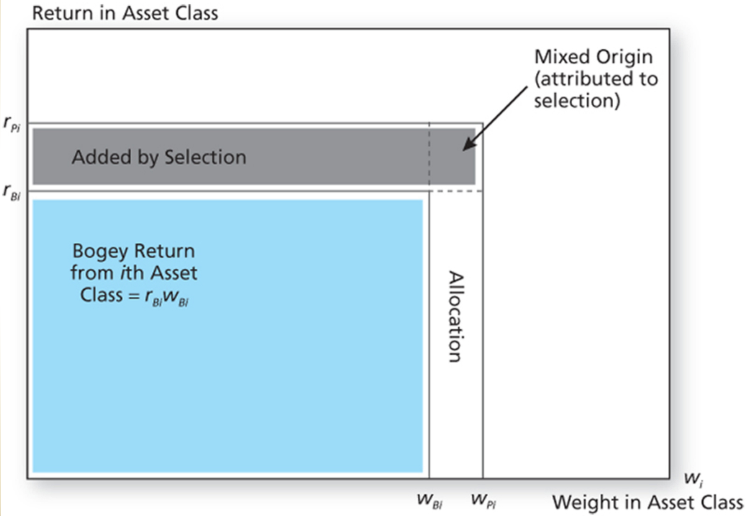


Table 24.6 Performance of the Managed Portfolio

Table 24.6

Performance of the managed portfolio

Component	Bogey Performance and Excess Return: Benchmark Weight	Bogey Performance and Excess Return: Return of Index during Month (%)
Equity (S&P 500)	.60	5.81
Bonds (Barclays Aggregate Index)	.30	1.45
Cash (money market)	.10	0.48
Bogey = (.60 × 5.81) + (.30 × 1.45) + (.10 × 0.48) = 3.97%		
Return of managed portfolio	5.34%	
− Return of bogey portfolio	3.97	
Excess return of managed portfolio	1.37%	

Performance Attribution

- Superior performance is achieved by:
  - overweighting assets in markets that perform well
  - underweighting assets in poorly performing markets

Table 24.7 Performance Attribution

Table 24.7

Performance attribution

A. Contribution of Asset Allocation to Performance

Market	(1) Actual Weight in Market	(2) Benchmark Weight in Market	(3) Active or Excess Weight	(4) Market Return (%)	(5) = (3) × (4) Contribution to Performance (%)
Equity	.70	.60	.10	5.81	.5810
Fixed-income	.07	.30	−.23	1.45	−.3335
Cash	.23	.10	.13	.48	.0624
Contribution of asset allocation					.3099

B. Contribution of Selection to Total Performance

Market	(1) Portfolio Performance (%)	(2) Index Performance (%)	(3) Excess Performance (%)	(4) Portfolio Weight	(5) = (3) × (4) Contribution (%)
Equity	7.28	5.81	1.47	.70	1.03
Fixed-income	1.89	1.45	0.44	.07	0.03
Contribution of selection within markets					1.06

Sector and Security Selection Decisions

- Good performance (a positive contribution) derives from overweighting high-performing sectors
- Good performance also derives from underweighting poorly performing sectors.

Table 24.8 Sector Selection within Equities

Table 24.8

Sector selection within the equity market

	(1)	(2)	(3)	(4)	(5) = (3) × (4)
	Beginning of Month Weights (%)				
Sector	Portfolio	S&P 500	Active Weights (%)	Sector Return (%)	Sector Allocation Contribution
Basic materials	1.96	8.3	−6.34	6.9	−0.4375
Business services	7.84	4.1	3.74	7.0	0.2618
Capital goods	1.87	7.8	−5.93	4.1	−0.2431
Consumer cyclical	8.47	12.5	−4.03	8.8	0.3546
Consumer noncyclical	40.37	20.4	19.97	10.0	1.9970
Credit sensitive	24.01	21.8	2.21	5.0	0.1105
Energy	13.53	14.2	−0.67	2.6	−0.0174
Technology	1.95	10.9	−8.95	0.3	−0.0269
TOTAL					1.2898

Table 24.9 Summing Up Component Contributions

Table 24.9

Portfolio attribution: summary

		Contribution (basis points)
1. Asset allocation		31
2. Selection		
a. Equity excess return (basis points)		
i. Sector allocation	129	
ii. Security selection	18	
	147 × .70 (portfolio weight) =	102.9
b. Fixed-income excess return	44 × .07 (portfolio weight) =	3.1
Total excess return of portfolio		137.0