Disagreement about Inflation and the Yield Curve*

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Abstract

We show that inflation disagreement, not just expected inflation, has a first-order effect on nominal interest rates. In contrast to expected inflation, which mainly affects the wedge between real and nominal yields, inflation disagreement affects nominal yields predominantly through its impact on the real side of the economy. We show theoretically and empirically that inflation disagreement raises real and nominal yields and their volatilities. Inflation disagreement is positively related to consumers' cross-sectional consumption growth volatility and trading in fixed income securities. Calibrating our model to disagreement, inflation, and yields reproduces the economically significant impact of inflation disagreement on yield curves.

Keywords: Inflation disagreement, relative entropy, real and nominal yields, yield volatilities, cross-sectional consumption growth volatility, trading on inflation beliefs

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Inflation expectations affect consumption and investment decisions and are important in determining nominal interest rates. Likewise, central banks base decisions about short-term interest rate changes on their inflation views. Inflation expectations and the compensation for inflation risk are also important drivers of long-term borrowing costs for households, firms, and governments. However, not everyone has the same expectation about inflation as the early work of Mankiw, Reis, and Wolfers (2004) shows. For example in December 2015, the interquartile range of annual inflation expectations is 0.9% to 4.6%, according to the Michigan Surveys of Consumers, and 1.87% to 2.25%, according to the Survey of Professional Forecasters.

Inflation disagreement leads to different investment and consumption decisions. For example, Malmendier and Nagel (2015) show that households who think that inflation will be high are more likely to borrow using fixed-rate mortgages and less likely to invest in long-term bonds.[1] Professional investors struggle with their inflation views too. For example, PIMCO's Total Return Fund shunned nominal U.S. treasuries after the Great Recession to bet on increased inflation which never materialized.[2] Given the evidence that households and professionals have different views about inflation and the important role that inflation plays for fixed income investments, we study how inflation disagreement affects interest rates and risk sharing.

We show, theoretically and empirically, that inflation disagreement, and not just expected inflation, has a first-order effect on nominal interest rates. Furthermore, the effect through which inflation disagreement operates is different than that for expected inflation. While expected inflation mainly impacts the wedge between real and nominal interest rates, inflation disagreement predominantly works through the real side of the economy. The mechanism is as follows. When investors disagree about inflation, they differ in their consumption-savings decisions because they perceive different real returns on investments. Hence, they take different positions in inflation-sensitive securities. For instance, consider two otherwise identical investors with different views about long-term inflation. In equilibrium, the investor who thinks inflation will be high will buy Treasury inflation protected securities or chose a fixed rate mortgage whereas an investors with the opposite view will buy nominal Treasury bonds or borrow at floating rates. If inflation turns out to be high, then the investor who thought inflation will be low loses wealth relative to the other investor. Ex-ante, each investor expects to capture wealth from the other investor and, hence, they expect future

[1] Piazzesi and Schneider (2012) show that inflation disagreement among younger and older households may reconcile their different investment decisions. Doepke and Schneider (2006) show that even moderate inflation episodes lead to sizable wealth redistributions of U.S. households.

[2] See, for example, thereformedbroker.com/2014/09/28/do-we-need-to-fire-pimco/.

consumption to be higher than without disagreement. When the income effect dominates the substitution effect, then the desire to consume more today drives an increased demand for borrowing. If aggregate consumption cannot fully adjust to the increased consumption demand, then the real interest rate increases for markets to clear.

We formalize this intuition in a tractable model with transparent economic forces. Specifically, we assume complete markets in a pure exchange economy where two investors differ in their beliefs about the distribution of inflation, not just expected inflation.[3] There are no frictions and, thus, inflation has no effect on real quantities when there is no inflation disagreement. In this case, money is neutral and expected inflation has a well-known one-to-one effect on nominal yields. In contrast, inflation disagreement leads to a feedback into the real economy which increases the cross-sectional consumption volatility and the level and volatility of real yields.[4] We show that this feedback effect, induced by heterogeneity in investor's consumption and investment decisions driven by their different inflation views, unambiguously increases nominal interest rates even though the effects of inflation disagreement on the inflation risk premium are ambiguous.

Empirically, we find that inflation disagreement has a strong impact on the nominal yield curve. We use the Surveys of Consumers from the University of Michigan (MSC) and the Survey of Professional Forecasters (SPF) to compute disagreement measures about one-year expected inflation for households and professionals, respectively. These surveys differ with respect to the sophistication of the constituency, the size of the survey, and the data frequency; thus they provide complementary support for our predictions. We show that there exists a statistically and economically positive relation between inflation disagreement and nominal yields across all maturities after controlling for expected inflation. Specifically, an increase in disagreement of households/professionals by one standard deviation (1.6%/0.3%) raises nominal yields on average by 54%/36% of their standard deviations. Inflation disagreement remains economically and statistically significant when controlling for expected inflation and inflation volatility. Moreover, the volatilities of nominal yields increase with inflation disagreement and the coefficient estimates also have large economic significance.

Our empirical results show that inflation disagreement has a first-order effect on nominal interest rates that is distinctly different from the effect of expected inflation. This is not surprising, given that the channel through which expected inflation mainly operates is different too. To empirically verify the economic channel through which inflation disagree-

[3]The economic mechanism that increases interest rates also works in a production economy with positive capital adjustment costs.

[4]The real yield is decreasing with inflation disagreement, if the substitution effect dominates the income effect, which requires a risk aversion coefficient less than one with power utility.

ment impacts nominal yields, we consider different proxies for the real yield and show that inflation disagreement has an economically and statistically positive effect on real yields.[5] Specifically, using real yield data from Chernov and Mueller (2012), we find that an increase in disagreement of households/professionals by one standard deviation raises real yields on average by 58%/39% of their standard deviation. In addition, consistent with our theory, real yield volatilities increase with inflation disagreement and the coefficient estimates also have large economic significance.

In our model, inflation disagreement affects yields because it leads to heterogeneity in consumption and investment decisions. Hence, to empirically test the economic channel through which inflation disagreement operates, we verify, using the Consumer Expenditure Survey (CEX), that there is indeed a positive relation between cross-sectional consumption growth volatility and inflation disagreement. We also show that inflation disagreement has a statistically positive effect on trading in nominal Treasury bonds, fixed income futures, and inflation swaps. These securities have a significant inflation exposure and, thus, investors may use them to directly trade on their inflation beliefs.[6] Moreover, this evidence alleviates the concern that inflation disagreement impacts yields because of its correlation with disagreement about other economic quantities such as GDP growth or earnings.[7] To conclude, the fact that inflation disagreement is positively related with the level and volatility of real yields, the cross-sectional consumption growth volatility, and trading in fixed income securities including inflation swaps makes it unlikely that inflation disagreement does not operate through our economic channel and unambiguously raises nominal yields.

Our theory predicts that disagreement about the distribution of inflation, not just expected inflation, raises real and nominal yields too. To test this prediction, we use the probability distribution forecasts for one-year inflation rates from the SPF to calculate disagreement about the variance and skewness of inflation. We find that there is an economically and statistically positive relation between real and nominal yields and disagreement about the variance and skewness of inflation.

A large literature in economics and finance uses inflation disagreement as a measure of inflation uncertainty, or more generally, economic uncertainty.[8] However, there is no clear

[5]We show in the Internet Appendix that inflation disagreement also has an economically and statistically positive effect on the break-even inflation rate and the inflation risk premium.

[6]Armantier, de Bruin, Topa, van der Klaauw, and Zafar (2015) show that consumers act on the inflation expectations they report in the MSC.

[7]We also show in the Internet Appendix that real and nominal yields and their volatilities are higher when inflation disagreement is high after controlling for disagreement about real GDP growth and earnings.

[8]For example, Bloom (2009) and Wright (2011) use disagreement among forecasters as a measure of uncertainty and Ilut and Schneider (2014) use disagreement as a measure of uncertainty aversion.

theoretical link between disagreement and uncertainty. Further, the empirical support for this assumption is mixed.[9] Still, to address the concern that economic uncertainty, not inflation disagreement, could be driving our results, we show that all our empirical findings are robust to controlling for inflation volatility. Moreover, we show that the impact of inflation disagreement on yields is robust to including five different measures of economic uncertainty (real consumption growth volatility, real GDP growth volatility, industrial production growth volatility, the Jurado, Ludvigson, and Ng (2015) Uncertainty Measure, and the Baker, Bloom, and Davis (2015) Uncertainty Measure).

In the final part of the paper, we show, by imposing more structure on our model, that it quantitatively matches our empirical results. Specifically, we consider two investors who disagree about the dynamics of expected inflation and are endowed with habit-forming preferences which helps to match asset pricing moments. The model admits closed-form solutions for bond prices, is rich enough to capture average yields and yield volatilities, and generates upward sloping real and nominal yield curves. We calibrate the model to the data by matching the average and volatility of inflation disagreement and the mean and volatility of consensus inflation in the SPF. The calibrated model shows that inflation disagreement has a first-order effect on real and nominal yields and their volatilities with a plausible risk premium and Sharpe ratio for inflation risk. Moreover, performing our main empirical tests on simulated data leads to statistical and economic significance of inflation disagreement that is consistent with the data.

Our paper is part of a growing literature that studies how disagreement impacts bond markets.[10] Xiong and Yan (2010) show that a moderate amount of heterogeneous expectations about inflation can quantitatively explain bond yield volatilities, the failure of the expectations hypothesis, and the Cochrane and Piazzesi (2005) forward factor predictability. Buraschi and Whelan (2013) and Whelan (2014) use survey data about various macroeconomic quantities to study the effects of disagreement on yield curve properties. Hong, Sraer, and Yu (2014) study how disagreement about expected inflation interacted with short-sale constraints can impact the pricing of long maturity bonds. Giacoletti, Laursen, and Singleton (2015) study the impact of yield disagreement in a dynamic arbitrage-free term structure model. Our paper differs from all of these works as we derive novel theoretical predictions

[9] Figure 17.1 in Zarnowitz (1992) shows simple examples of distributions where high and low disagreement is associated with either high or low uncertainty. While some papers empirically show that there is a very high correlation between inflation disagreement and measures of economic uncertainty, justifying the use of inflation disagreement as measure for economic uncertainty, other works argue that inflation disagreement is distinctly different from inflation uncertainty and other forms of economic uncertainty.

[10] Other papers that empirically explore the role of inflation beliefs on the term structure include Ang, Bekaert, and Wei (2007), Adrian and Wu (2010), Chun (2011), and Chernov and Mueller (2012).

that we empirically test on quantities including real and nominal yield levels, their volatilities, and the cross-sectional consumption growth volatility. Another aspect of our work that differs from the literature is that we calibrate our model to disagreement data.

This paper is also part of the large literature on heterogeneous beliefs models.[11] Our contribution to this literature is threefold. First, we provide novel predictions for the effects of inflation disagreement on interest rates. Second, our analysis is not limited to a tightly parameterized inflation disagreement model. Third, we provide a methodological contribution to the literature which can be applied to similar settings in the future, that does not rely on continuous-time finance techniques and, hence, is accessible to a broader audience.

1 Theoretical Results

This section provides a general framework to study the impact of inflation disagreement on interest rates.

Our model is a pure exchange economy with a single perishable consumption good. The time horizon T' of the economy can be finite or infinite. Real prices are measured in units of the consumption good and nominal prices are quoted in dollars. Let C_t denote the exogenous real aggregate consumption process and Π_t the exogenous price process that converts real prices into nominal prices, that is, nominal consumption is $\Pi_t C_t$. The sample space Ω and the information set $\mathcal{F}_t$ on which we define all random variables and probability measures, in short beliefs, represent the uncertainty in the economy.

Two investors share a common subjective discount factor ρ, a Bernoulli utility function $u(C/H) = \frac{1}{1-\gamma}(C/H)^{1-\gamma}$ with $\gamma > 0$, and an exogenous habit process or, more generally, a preference shock H_t. Let $\mathbb{P}^i$ denote investor i's belief about inflation Π_t, consumption C_t, and the preference shock H_t. The investors have the same information set $\mathcal{F}_t$ and agree on the events of $\mathcal{F}_t$ that cannot occur. Hence, there is no asymmetric information and the likelihood ratio defined as $\lambda_t \equiv \frac{d\mathbb{P}^2}{d\mathbb{P}^1}$ is strictly positive and finite.

Both investors trade a complete set of Arrow-Debreu (AD) securities. There is a unique equilibrium AD pricing functional that both investors agree on and that will be determined in Proposition 1. Let ξ_t^i denote the state price density that represents the AD pricing functional

[11] See for example Harris and Raviv (1993), Detemple and Murthy (1994), Zapatero (1998), Basak (2000), Yan (2008), Gallmeyer and Hollifield (2008), Dumas, Kurshev, and Uppal (2009), Cvitanić, Jouini, Malamud, and Napp (2012), Chen, Joslin, and Tran (2010, 2012), and Bhamra and Uppal (2014). Basak (2005) provides a survey of this literature.

under the probability measure $\mathbb{P}^i$ and $\mathbb{E}^i$ the expectation under $\mathbb{P}^i$. Each investor chooses a consumption process C_t^i to maximize

$$\mathbb{E}^i\left[\sum_{t=0}^{T'} e^{-\rho t} u\left(\frac{C_t^i}{H_t}\right)\right] \qquad \text{s.t.} \qquad \mathbb{E}^i\left[\sum_{t=0}^{T'} \xi_t^i C_t^i \, dt\right] \leq w_0^i, \tag{1.1}$$

where w_0^i denotes initial wealth of investor i.[12] If time is continuous, then replace the sums in equation (1.1) with integrals.

To focus on inflation disagreement, we make the following assumption.

Assumption 1 (Inflation Disagreement)**.** *Investors only disagree about the distribution of inflation.*

While this assumption rules out disagreement about the correlation between consumption and inflation, it does not imply that inflation is independent of consumption and the habit. Moreover, it allows for disagreement about higher order moments of inflation, not just expected inflation.[13]

We determine the equilibrium consumption allocations C_t^1 and C_t^2 and state price densities ξ_t^1 and ξ_t^2 in the next proposition.

Proposition 1 (Consumption Allocations and State Price Densities)**.** *Optimal consumption allocations are* $C_t^1 = f(\lambda_t)C_t$ *and* $C_t^2 = (1 - f(\lambda_t))C_t$ *with*

$$f(\lambda_t) = \frac{1}{1 + (y\lambda_t)^{\frac{1}{\gamma}}}, \tag{1.2}$$

where $y = \frac{y^2}{y^1}$ *and* y^i *is the constant Lagrange multiplier from the static budget constraint given in equation (1.1). The state price densities are*

$$\xi_t^1 = (y^1)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} f(\lambda_t)^{-\gamma}, \qquad \xi_t^2 = (y^2)^{-1} e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1} (1 - f(\lambda_t))^{-\gamma}. \tag{1.3}$$

The likelihood ratio λ_t summarizes the impact of disagreement on the consumption allocations and state prices. To derive the equilibrium in Proposition 1, we do not impose

[12] Investors are either endowed with shares of a claim on aggregate consumption or with a fraction of the aggregate consumption process.

[13] We provide proofs for all results using a weaker assumption where investors can disagree about the joint distribution of inflation, consumption, and the preference shock. For instance, investors can disagree on the correlation between output and inflation as long as they agree on the joint distribution of consumption and the preference shock.

any restrictions on the likelihood ratio λ_t. It can be driven by a Brownian motion or a Poisson process where only one of the investors will survive in the long run. It can also be driven by a bounded martingale to guarantee the survival of all investors in the long run.[14]

Example 1. Edgeworth Box: *The left plot of Figure 1 presents an Edgeworth box example with two dates* 0 *and* 1*. For simplicity, we set the subjective discount factor to zero and normalize aggregate consumption and the habit or preference shock to one. The price level today is normalized to one and the price level tomorrow is either* Π_u *or* Π_d*. There are two investors with different beliefs* $\mathbb{P}^i = (p^i, 1-p^i)$*. The likelihood ratio* λ *equals* $\frac{p^2}{p^1}$ *with probability* p^1 *and* $\frac{1-p^2}{1-p^1}$ *with probability* $1-p^1$*. Define the disagreement parameter as* $\Delta = \frac{p^2-p^1}{p_1}$.[15] *The baseline parameters for the Edgeworth box example are* $p^1 = 0.4, p^2 = 0.6, \Pi_u = 1.25,$ *and* $\Pi_d = 0.9$*.*

Since there is no uncertainty about consumption in this example, full insurance is Pareto efficient if there is no disagreement about inflation ($\lambda_u = \lambda_d = 1$). Hence, each investor consumes the same share of consumption in the high and low inflation state in equilibrium. Suppose investors are endowed with 0.5 units of the date zero consumption good in both states. Then, the initial endowment is an equilibrium if there is no disagreement (the tangency point of the blue solid indifference curves). This is no longer true when investors disagree about inflation. For instance, if the first investor thinks that the low inflation state is more likely, then she consumes a larger fraction of consumption in this state because $\lambda_u > \lambda_d$ and, thus, $f_u < f_d$. Therefore, full insurance is no longer an equilibrium and disagreement affects state prices. The tangency point of the red dashed indifference curves in the left plot of Figure 1 denotes the equilibrium consumption allocation for this case.

We consider two additional examples, where the economy is dynamic and the likelihood ratio is unbounded, to illustrate the generality of our results. In both examples, the consumption and habit process are normalized to one. The three examples allow us to focus on how inflation disagreement impacts real and nominal bonds because $\xi_t^1 = \xi_t^2 = 1$ if there is no disagreement.

Example 2. Geometric Brownian Motion: *Consider a continuous-time economy in which the price level* Π_t *follows a geometric Brownian motion and two investors disagree on*

[14] See Basak (2005) and the references therein for a discussion of heterogenous beliefs models when the likelihood ratio is driven by Brownian motions and there is effectively only disagreement about means. For details on investors' survival in heterogenous beliefs models see Fedyk, Heyerdahl-Larsen, and Walden (2013) and the references therein.

[15] We divide by p_1 to make the disagreement parameter comparable across examples.

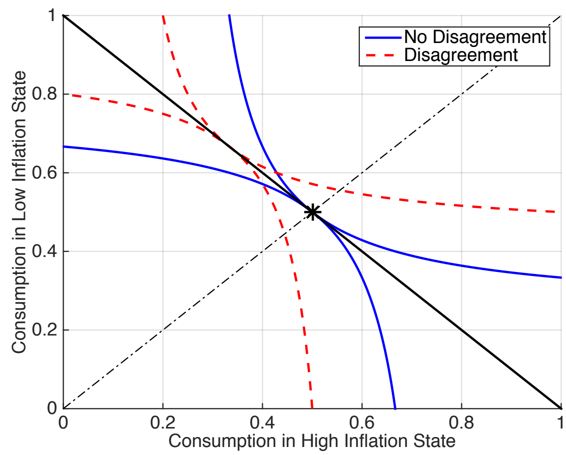

Figure 1: Edgeworth Box with Inflation Disagreement

This figure shows an Edgeworth box example when both investors are endowed with 0.5 units of the date zero consumption good in both states and $\gamma = 2$. The red dashed indifference curves are tangent at the equilibrium allocation with disagreement and the blue solid indifference curves are tangent at the equilibrium allocation without disagreement. Full insurance is no longer an equilibrium when there is disagreement.

the expected inflation rate. The dynamics of the price level are

$$d\Pi_t = x^i \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_t^i, \tag{1.4}$$

where x^i *denotes the expected inflation rate and* z_t^i *denotes the perceived nominal shock of investor* i*. The dynamics of the likelihood ratio* λ_t *are*

$$d\lambda_t = \Delta \lambda_t \, dz_t^1, \qquad \Delta = \frac{x^2 - x^1}{\sigma_\Pi}. \tag{1.5}$$

The baseline parameters for the GBM example are $\sigma_\Pi = 2\%$, $x^1 = 1.5\%$, *and* $x^2 = 2.5\%$.

Example 3. Poisson Process: *Consider a continuous-time economy in which the dynamics of the price level are*

$$d\Pi_t = x\Pi_{t-} \, dt + \theta \Pi_{t-} dN_{t-}^i, \tag{1.6}$$

where x *denotes a constant and* θ *denotes the constant jump size with* $\theta \neq 0$ *and* $\theta > -1$*. The two investors agree on the jump times of the Poisson process but disagree on the jump intensity* l^i*. Hence, they disagree on the expected inflation rate* $x + \theta l^i$*. The dynamics of the* *likelihood ratio* λ_t *are*

$$d\lambda_t = \Delta\lambda_{t-}\left(dN^1_{t-} - l^1 dt\right), \qquad \Delta = \frac{l^2 - l^1}{l^1}. \tag{1.7}$$

The baseline parameters for the Poisson example are $x = 6\%, \theta = -10\%, l^1 = 12.5\%$, *and* $l^2 = 27.5\%$.

The Edgeworth box example is simple and transparent and allows us to illustrate the effects of inflation disagreement without relying on continuous-time finance techniques. The GBM example, where (log) inflation rates are normally distributed with a constant mean and volatility, focuses on the effects of disagreement about expected inflation on consumption allocations and asset prices. The Poisson example illustrates how disagreement about expected inflation and higher-order moments of inflation affect consumption allocations and asset prices. The three examples also illustrate that we do not impose any restrictions on the likelihood ratio. Specifically, the likelihood ratio is a bounded martingale in the Edgeworth box example, a martingale with unbounded variation in the GBM example, and a martingale with finite variation in the Poisson example.

1.1 Definitions

All bonds are default-free zero-coupon bonds, in zero-net supply, and are priced using the state price densities from Proposition 1. A nominal bond pays one dollar at its maturity. Let $P_{t,T}$ denote the nominal price of a nominal bond maturing at T with continuously-compounded nominal yield $y^P_{t,T} = -\frac{1}{T-t}\log(P_{t,T})$, where $T \in [t, T']$. The nominal price of a nominal bond is

$$P_{t,T} = \mathbb{E}^i_t\left[\frac{\xi^i_T}{\xi^i_t}\frac{\Pi_t}{\Pi_T}\right]. \tag{1.8}$$

A real bond pays one unit of the consumption good at its maturity. Let $B_{t,T}$ denote the real price of a real bond maturing at T with continuously-compounded real yield $y^B_{t,T} = -\frac{1}{T-t}\log(B_{t,T})$, where $T \in [t, T']$. The real price of a real bond is

$$B_{t,T} = \mathbb{E}^i_t\left[\frac{\xi^i_T}{\xi^i_t}\right]. \tag{1.9}$$

Let $\mathrm{RX}_{t,T}$ denote the real gross return on a nominal bond in excess of the real gross return on a real bond both maturing at T:

$$\mathrm{RX}_{t,T} = \left(\frac{P_{T,T}}{\Pi_T} \Big/ \frac{P_{t,T}}{\Pi_t} \right) \Big/ \frac{B_{T,T}}{B_{t,T}} = e^{(y^P_{t,T} - y^B_{t,T})(T-t)} \frac{\Pi_t}{\Pi_T}. \tag{1.10}$$

The nominal return on a nominal bond is certain, while the real return on a nominal bond is a bet on inflation risk, that is, a bet on the real value of one dollar which is $\frac{1}{\Pi_T}$. The inflation risk premium perceived by investor i is defined as the expected real excess return on a nominal bond maturing at T:

$$\mathbb{E}^i_t \left[\mathrm{RX}_{t,T}\right] = e^{(y^P_{t,T} - y^B_{t,T})(T-t)} \mathbb{E}^i_t \left[\frac{\Pi_t}{\Pi_T} \right], \qquad i = 1, 2. \tag{1.11}$$

Let $\mathrm{IRP}^i_{t,T}$ denote the annualized log inflation risk premium and $\mathrm{EINFL}^i_{t,T}$ the annualized expected log inflation rate perceived by investor i:[16]

$$\mathrm{EINFL}^i_{t,T} \equiv -\frac{1}{T-t} \log \left(\mathbb{E}^i_t \left[\frac{\Pi_t}{\Pi_T} \right] \right), \tag{1.12}$$

$$\mathrm{IRP}^i_{t,T} \equiv \frac{1}{T-t} \log \left(\mathbb{E}^i_t \left[\mathrm{RX}_{t,T}\right] \right) = y^P_{t,T} - y^B_{t,T} - \mathrm{EINFL}^i_{t,T}. \tag{1.13}$$

Hence, the nominal bond yield is

$$y^P_{t,T} = y^B_{t,T} + \mathrm{EINFL}^i_{t,T} + \mathrm{IRP}^i_{t,T}, \qquad i = 1, 2. \tag{1.14}$$

Investors agree on prices, so they agree on the break-even inflation rate denoted by $\mathrm{BEIR}_{t,T} = y^P_{t,T} - y^B_{t,T}$ and, thus,

$$y^P_{t,T} = y^B_{t,T} + \mathrm{BEIR}_{t,T}. \tag{1.15}$$

Hence, inflation disagreement affects the nominal yield through two channels: (i) the real yield and (ii) the break-even inflation rate. We discuss the two channels in the remainder of this section.

[16] Jensen inequality implies that

$$\mathrm{EINFL}_{t,T} = -\frac{1}{T-t} \log \left(\mathbb{E}_t \left[\frac{\Pi_t}{\Pi_T} \right] \right) \leq \frac{1}{T-t} \mathbb{E}_t \left[\log \left(\frac{\Pi_T}{\Pi_t} \right) \right] \leq \frac{1}{T-t} \log \left(\mathbb{E}_t \left[\frac{\Pi_T}{\Pi_t} \right] \right),$$

and, thus, $\mathrm{IRP}_{t,T}$ is higher than the inflation risk premium implied by other measures for expected inflation.

1.2 Real Yields and the Cross-Sectional Consumption Volatility

We show in the next theorem how inflation disagreement affects the level and volatility of real yields.

Theorem 1 (Real Yields). *If Assumption 1 is satisfied, then*

1. *real yields and their volatilities do not depend on disagreement if* $\gamma = 1$*,*
2. *real yields are higher with disagreement if* $\gamma > 1$ *(the opposite is true if* $\gamma < 1$*), and*
3. *the volatility of real yields is higher with disagreement if* $\gamma \neq 1$ *and* λ_t *is independent of* C_t *and* H_t*.*

The left plot of Figure 2 shows real one-year yields as a function of γ. When there is no disagreement, then the real yield is zero (blue solid line). With disagreement, real yields are nonnegative if $\gamma \geq 1$ and negative otherwise. The red dashed, green dash-dotted, and solid black circle lines represent the baseline for the Edgeworth box, GBM, and Poisson examples. The black dashed circle line shows real yields with lower jump intensities ($l^1 = 5\%$ and $l^2 = 20\%$) and the black dash-dotted circle line shows real yields with higher jump intensities ($l^1 = 20\%$ and $l^2 = 35\%$) than in the baseline case. The three Poisson examples show that real yields are increasing in $\Delta = \frac{l^2 - l^1}{l^1}$ if $\gamma > 1$ and decreasing if $\gamma < 1$. We determine bond prices in all three examples in closed form.[17]

Why are real yields higher with disagreement if $\gamma > 1$ and lower if $\gamma < 1$? Intuitively, investors make different consumption and savings decisions based on their differing views about inflation. Both investors think they will capture consumption from the other investor in the future; hence, classical income and substitution effects impact the demand for consumption today. If $\gamma > 1$, then the real interest rate rises to counterbalance increased demand for borrowing. If $\gamma < 1$, then the real interest rate falls to counterbalance lowered demand for borrowing.[18] There is no effect on real yields if the income and substitution effects exactly offset ($\gamma = 1$), as in Xiong and Yan (2010).

When investors make different consumption and savings decisions based on their differing views about inflation, then individual consumption growth should be more volatile. Formally, the cross-sectional consumption growth variance from time t to T is

$$\sigma^2_{\mathrm{CS}}(\lambda_t, \lambda_T) = \frac{1}{4}\left(\log\left(\frac{C^1_T}{C^1_t}\right) - \log\left(\frac{C^2_T}{C^2_t}\right)\right)^2 = \frac{1}{4\gamma^2}\left(\log\left(\frac{\lambda_T}{\lambda_t}\right)\right)^2. \tag{1.16}$$

[17] We provide details in the Internet Appendix.

[18] See Epstein (1988) or Gallmeyer and Hollifield (2008) for additional details.

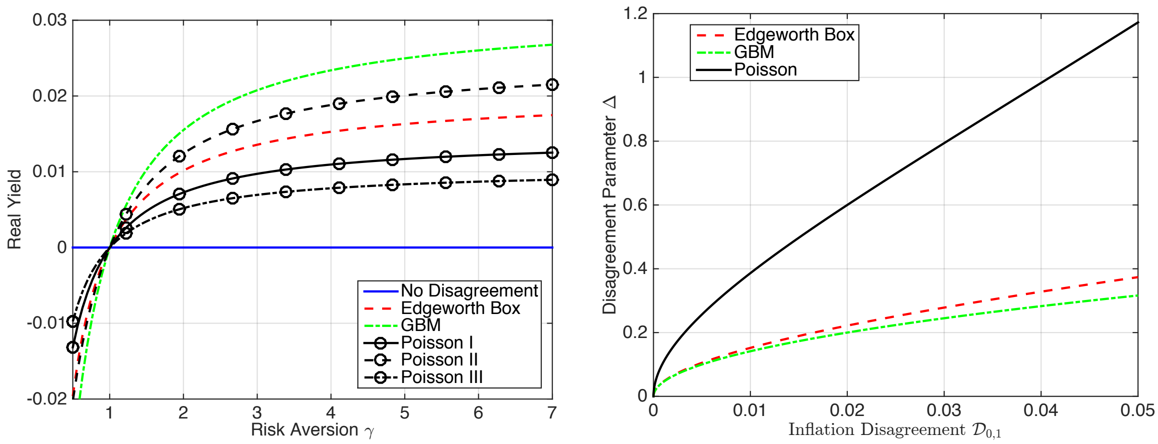

Figure 2: Disagreement Measure and Real Yields

The left plot shows real one-year yields as a function of γ. When there is no disagreement, then the real yield is zero (blue solid line). With disagreement, real yields are nonnegative if $\gamma \geq 1$ and negative otherwise. The right plot shows the disagreement parameter Δ in all three examples, using the baseline parameters, as an increasing function of the inflation disagreement measure $\mathcal{D}_{0,1}$.

There are no fluctuations in the cross-sectional consumption distribution when there is no disagreement ($\lambda_T = \lambda_t = 1$). Moreover, there is less variation in cross-sectional consumption allocations if investors are more risk averse because they trade less aggressively on their beliefs. Trading on beliefs not only increases the cross-sectional consumption growth volatility, but it also leads to more volatile real yields.

We generalize the real yield and cross-sectional consumption growth volatility results by defining a measure of disagreement to study the effects of changes in disagreement on real yield levels and the cross-sectional consumption growth volatility. Measuring disagreement is straightforward in all three examples because investors' beliefs belong to the same class of distributions and there is only disagreement about a single parameter. To measure disagreement among investors more generally, we define disagreement as relative entropy per year.[19] This measure allows us to study the effects of disagreement on bond yields when investors have beliefs that differ by more than one parameter or do not even belong to the same class of distributions.

Definition 1 (Inflation Disagreement Measure)**.** *Consider a belief structure* $\mathcal{B}_{t,T} = (\mathbb{P}^1, \mathbb{P}^2)$

[19] The relative entropy or Kullback-Leibler divergence is widely used in statistics and information theory to measure the difference between two probability distributions (see Kullback (1959)). While this measure is not symmetric, the results do not change if we compute the relative entropy with respect to the second investor. Similarly, all our results still follow if we consider other divergence measures suggested in the literature (see Csiszár and Shields (2004)).

with the likelihood ratio $\lambda_u = \frac{d\mathbb{P}^2}{d\mathbb{P}^1} \mid_{\mathcal{F}_u}$ *for all* $t \leq u \leq T$*. Define disagreement as*

$$\mathcal{D}_{t,T} = -\frac{1}{T-t}\mathbb{E}_t^1\left[\log\left(\frac{\lambda_T}{\lambda_t}\right)\right]. \tag{1.17}$$

Disagreement $\mathcal{D}_{t,T}$ is nonnegative. It is zero if and only if the two investors have the same belief, in which case $\lambda_t = \lambda_T = 1$. The right plot of Figure 2 shows the disagreement parameter Δ for all three examples as a function of $\mathcal{D}_{0,1}$. The red dashed line represents the Edgeworth box example with $\Delta = \frac{p^2-p^1}{p^1}$, the green dash-dotted line represents the Brownian example with $\Delta = \frac{x^2-x^1}{\sigma_\Pi}$, and the black solid line represents the Poisson example with $\Delta = \frac{l^2-l^1}{l^1}$. The plot shows that $\mathcal{D}_{0,1}$ strictly increases in the Δ's of all three examples and that it is zero if and only if $\Delta = 0$.

We show in the next theorem that all results of Theorem 1, except for the yield volatility result, generalize when we compare economies with differing levels of disagreement (holding everything else fixed including γ and ρ).

Theorem 2. *Adopt Assumption 1 and consider two economies* $\mathcal{E} = (\mathcal{B}_{t,T}, f(\lambda_t))$ *and* $\mathcal{E}_\eta = \left(\mathcal{B}^\eta_{t_\eta,T_\eta}, f(\eta_{t_\eta})\right)$ *with*

- *the same time horizon, that is,* $\tau = T_\eta - t_\eta = T - t$*,*
- *the same current consumption allocations, that is,* $f_t = f(\lambda_t) = f(\eta_{t_\eta})$*,*
- *the same distribution of real quantities, that is, the joint distribution of* $\frac{C_{T_\eta}}{C_{t_\eta}}$ *and* $\frac{H_{T_\eta}}{H_{t_\eta}}$ *conditional on* $\mathcal{F}_{t_\eta}$ *is equal to the joint distribution of* $\frac{C_T}{C_t}$ *and* $\frac{H_T}{H_t}$ *conditional on* $\mathcal{F}_t$*, and*
- λ_t *second-order stochastically dominates* η_{t_η}*.*[20]

Then, economy $\mathcal{E}_\eta$ *has more disagreement than economy* $\mathcal{E}$*, that is,* $\mathcal{D}^\eta_{t_\eta,t_\eta+\tau} \geq \mathcal{D}_{t,t+\tau}$*, and*

1. *real yields are the same in both economies if* $\gamma = 1$*,*
2. *real yields are higher in economy* $\mathcal{E}_\eta$ *than in economy* $\mathcal{E}$ *if* $\gamma > 1$ *(the opposite is true if* $\gamma < 1$*), and*
3. *the expected cross-sectional consumption growth volatility is higher in economy* $\mathcal{E}_\eta$ *than in economy* $\mathcal{E}$ *if* $\frac{\lambda_T}{\lambda_t}$ *and* ε *are independent.*

[20] See Definition 3 in the Appendix for a formal definition.

The concept of second-order stochastic dominance allows us to focus on one-dimensional decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$. This one-dimensional multiplicative decomposition nevertheless covers a large class of stochastic processes.[21] Intuitively, one can think of η_{T_η} as a noisy version of λ_T. For instance, λ_t second-order stochastic dominates η_t in all three examples if $\Delta_\eta \geq \Delta$ and, thus, real yields and the expected cross-sectional consumption growth volatility are increasing functions of inflation disagreement as shown in Figure 3.

The left plot of Figure 4 shows that real yield volatility is also increasing in disagreement. The black star and black diamond lines represent the Poisson example with $\gamma = 2$ and $\gamma = 0.5$, respectively. The green dash-dotted star and the green dash-dotted diamond lines represent the GBM example with $\gamma = 2$ and $\gamma = 0.5$, respectively. Real yield volatility in the GBM and Poisson example is higher for $\gamma = 0.5$ than for $\gamma = 2$ since the expected cross-sectional consumption growth volatility is decreasing with risk aversion.

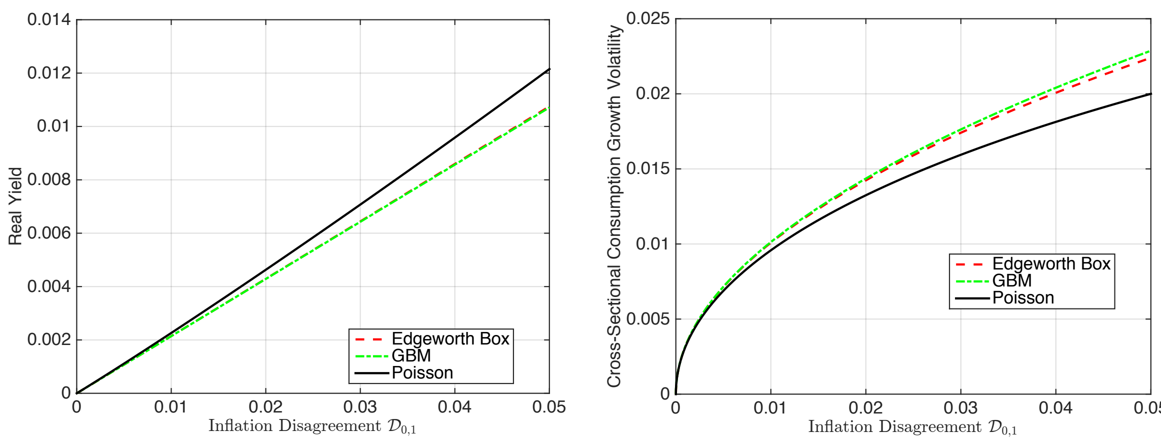

Figure 3: Real Yields and Consumption Volatility

The left plot shows that real yields are increasing in inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma = 7$. The right plot shows that the expected cross-sectional consumption growth volatility is strictly increasing in inflation disagreement $\mathcal{D}_{0,1}$.

1.3 Nominal Yields

We already know how expected inflation affects nominal yields and, thus, we fix the market view or belief about expected inflation to provide a meaningful comparison between nominal

[21] All results still follow if we consider additive mean-independent and comonotone decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$.

yields with and without disagreement. Before we define and discuss the market view or belief about the expected real value of one dollar, recall the decomposition of nominal bond yields:

$$y^P_{t,T} = y^B_{t,T} + \mathrm{BEIR}_{t,T} = y^B_{t,T} + \mathrm{EINFL}^i_{t,T} + \mathrm{IRP}^i_{t,T}, \qquad i = 1, 2. \tag{1.18}$$

Investors agree on the real yield and the break-even inflation rate, but they may have different beliefs about inflation and the compensation for inflation risk. If they disagree about the expected real value of one dollar, then by equation (1.18) they disagree on the inflation risk premium. For example, consider the case when the first investor predicts lower inflation than the second investor, that is, $\mathrm{EINFL}^1_{t,T} < \mathrm{EINFL}^2_{t,T}$. Subtracting the expected inflation rate from the agreed upon break-even inflation rate leads to a higher perceived compensation for inflation risk for the first investor, that is, $\mathrm{IRP}^1_{t,T} > \mathrm{IRP}^2_{t,T}$.[22]

If investors agree on the expected real value of one dollar, that is, $\mathrm{EINFL}^1_{t,T} = \mathrm{EINFL}^2_{t,T}$, then they agree on the inflation risk premium. Hence, the nominal yield is higher with disagreement if the real yield plus the inflation risk premium is higher with disagreement. However, if $\mathrm{EINFL}^1_{t,T} \neq \mathrm{EINFL}^2_{t,T}$, then inflation disagreement affects the nominal yield through three channels: (i) the real yield, (ii) perceived inflation risk premiums, and (iii) perceived expected inflation.

To study the effects of disagreement, rather than the effects of an overall change in the expected real value of one dollar, on the nominal yield, we hold the market view about the expected real value of one dollar constant. However, it is not obvious which belief to hold constant when increasing inflation disagreement in a heterogeneous beliefs economy. We could consider a mean-preserving spread while keeping the average belief about the expected real value of one dollar constant to unambiguously increase disagreement. Still, this does not take into account that the belief of a wealthier investor has a stronger impact on real and nominal yields than the belief of a poorer investor. Hence, to take into account that a wealthier investor has a larger impact on prices, we define the market view as the weighted average across each investor's expected real value of a dollar, where the weights are given by the fraction of output that each investors consumes $(f(\lambda_t), 1 - f(\lambda_t))$.

Definition 2 (Market View or Belief)**.** *Let $\mathbb{P}^0$ denote the market view that satisfies*

$$\mathbb{E}^0_t\left[\frac{\Pi_t}{\Pi_T}\right] = f(\lambda_t)\mathbb{E}^1_t\left[\frac{\Pi_t}{\Pi_T}\right] + (1 - f(\lambda_t))\mathbb{E}^2_t\left[\frac{\Pi_t}{\Pi_T}\right]. \tag{1.19}$$

[22] See Section 1.3 of the Internet Appendix for a detailed discussion of the inflation risk premium.

In the remainder of this section, we hold the market view about inflation fixed when we increase disagreement and thus any changes in the break-even inflation rate are due to changes in the inflation risk premium and not expected inflation. To simplify the analysis, we rule out any risk premia for inflation risk when there is no disagreement and, thus, we make the following assumption.[23]

Assumption 2. *Inflation Π_t is independent of consumption C_t and the habit H_t.*

We show in the next theorem that inflation disagreement has qualitatively the same effect on nominal yields as on real yields even though the effects on the inflation risk premium are ambiguous.

Theorem 3 (Nominal Yield)**.** *Fix the market view as in Definition 2 and suppose Assumptions 1 and 2 are satisfied, then*

1. *the break-even inflation rate and nominal yields do not depend on disagreement if $\gamma = 1$ and*
2. *nominal yields are higher with disagreement if $\gamma > 1$ (the opposite is true if $\gamma < 1$) even though the effects of inflation disagreement on the break-even inflation rate are ambiguous if $\gamma \neq 1$.*

The right plot of Figure 4 shows nominal one-year yields as a function of risk aversion γ. The red dashed circle, green dash-dotted circle, and black circle lines represent the Edgeworth box, GBM, and Poisson examples, respectively, when there is no disagreement. The corresponding lines without circles represent the examples when there is disagreement and the market view is fixed. The plot shows that in all three examples nominal yields are higher with disagreement than without disagreement if $\gamma > 1$ and lower if $\gamma < 1$.

We discuss the implications for nominal yields when the market view changes through the GBM example. Investors share aggregate consumption equally, that is, $f = 0.5$. The expected inflation rate is two percent $\bar{x} = 2\%$, if there is no disagreement in which case the nominal yield is 1.96% (green dash-dotted circle line). We consider three different cases with disagreement: (i) baseline with $x^1 = 1.5\%$ and $x^2 = 2.5\%$ (green dash-dotted line), (ii) $x^1 = 1\%$ and $x^2 = 2\%$ (green dash-dotted plus line), and (iii) $x^1 = 2\%$ and $x^2 = 3\%$ (green dash-dotted cross line). The consumption share weighted-average belief in the first case is

[23] We relax this assumption in the Appendix, where we also allow for disagreement about the joint distribution of inflation, consumption, and the habit.

approximately 2% and, thus, the market view is the same with and without disagreement.[24] If the consumption share weighted-average belief is below 2%, then inflation disagreement lowers nominal yields if $\gamma < 1$, but does not always increase nominal yields if $\gamma > 1$. Intuitively, inflation disagreement pushes up real yields, but lowers the expected inflation rate. If the second effect dominates the first, then nominal yields are lower than in the no disagreement economy. The intuition is similar for the third case.

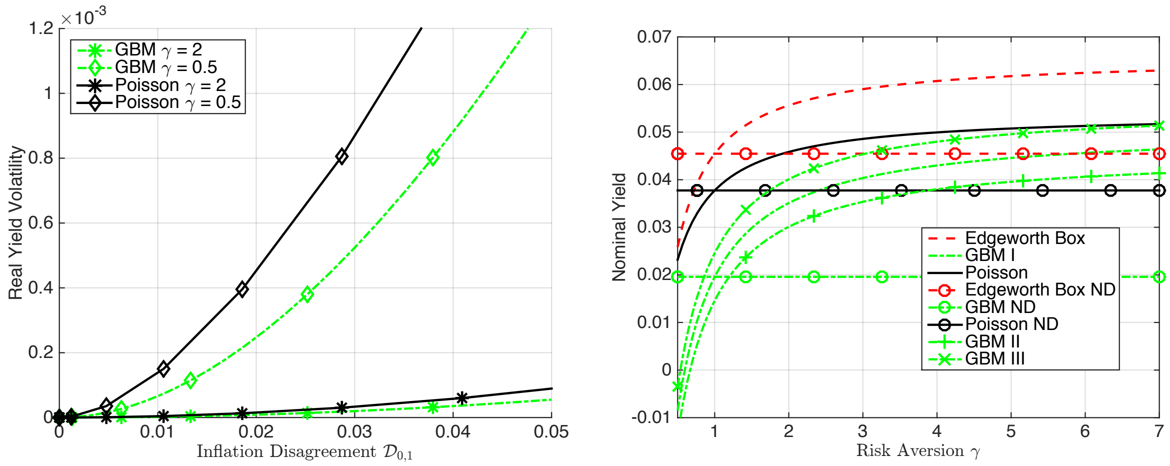

Figure 4: Real Yield Volatility and Nominal Yields

The left plot shows that the volatility of real yields is strictly increasing in inflation disagreement $\mathcal{D}_{0,1}$. The expected cross-sectional consumption growth volatility is decreasing in risk aversion and, thus, real yield volatility is lower with $\gamma = 2$ than with $\gamma = 0.5$. The right plot shows nominal one-year yields as a function of risk aversion. Nominal yields are higher (lower) with than without disagreement when $\gamma \geq (\leq) 1$ except for the cases GBM II and III, where the market view is not fixed.

The left plot of Figure 5 shows the difference between the break-even inflation rate in an economy with and without disagreement as a function of risk aversion. If $\gamma = 1$, then the break-even inflation rate does not depend on disagreement. Thus, the red dashed lines (Edgeworth Box example), the green dash-dotted lines (GBM example), and the black lines (Poisson example) all intersect at zero. If $\gamma > 1$, then the break-even inflation rate is higher with disagreement in the GBM and Poisson examples. The quantitative effect is smaller for the short-end of the yield curve and it is larger in the Poisson example than the GBM example. In contrast to real yields, the effects of inflation disagreement on the break-even inflation rate are ambiguous. For instance, consider an Edgeworth box example where risk aversion is greater than one and the second investor thinks that the high and low inflation state are equally likely. If the first investor thinks that the high inflation state is less

[24] In this example, we have that $0.5e^{-1.5\%} + 0.5e^{-2.5\%} \approx e^{-2\%}$.

likely (red dashed star line), than the break-even inflation rate is lower with than without disagreement. The opposite is true when the first investor thinks that the high inflation state is more likely (red dashed diamond line).

Nominal yields are always higher with inflation disagreement when $\gamma > 1$ even though the speculative trade induced by disagreement may lead to a lower inflation risk premium and, thus, a lower break-even inflation rate because the market view about expected inflation is fixed. The right plot of Figure 5 shows that nominal yields in all three examples are strictly increasing in inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma > 1$ and while keeping the consumption-share weighted expected value of one dollar fixed.

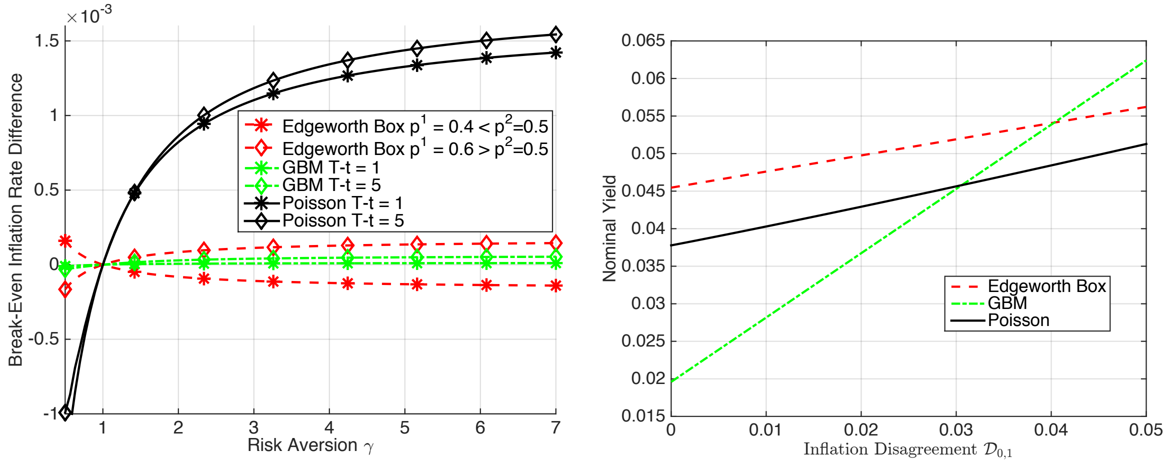

Figure 5: Break-Even Inflation and Nominal Yields

The left plot shows the difference between the break-even inflation rate in an economy with and without disagreement as a function of risk aversion γ. If $\gamma \geq 1$, then the break-even inflation rate is higher with disagreement in the GBM, Poisson, and second Edgeworth box examples. The opposite is true in the first Edgeworth box example. The right plot shows, using the baseline parameters of all three examples, the nominal one-year yield as an increasing function of inflation disagreement $\mathcal{D}_{0,1}$ when $\gamma = 7$ and the consumption-share weighted belief of both investors about the expected real value of one Dollar does not change with disagreement.

2 Empirical Evidence

To validate the theory, we use the Survey of Professional Forecasters (SPF) and the Michigan Surveys of Consumers (MSC) to empirically test whether disagreement about expected inflation affects nominal yields (Table 3), real yields (Table 4), real and nominal yield volatilities (Table 5), the cross-sectional consumption growth volatility (Table 6), and trading on inflation disagreement (Table 6). We also use the SPF to examine whether disagreement about the variance and skewness of inflation affects real and nominal yield levels (Tables 7 and 8). The two surveys differ with respect to the sophistication of their constituencies, the survey size, and the data frequency. Thus, they provide complementary support for our predictions.

2.1 Data

Inflation Disagreement. Disagreement about inflation, our main explanatory variable, is the cross-sectional standard deviation of one year ahead inflation forecasts abbreviated as DisInf. Disagreement of consumers is directly taken from the MSC database (Table 32: Expected Change in Prices During the Next Year) and disagreement of professionals is computed from their individual responses for the CPI Inflation Rate taken from the SPF database (series CPI).[25] To compute disagreement about the variance and skewness of one-year inflation rates, we use GDP deflator forecasts of professionals, which, in contrast to the MSC and the SPF for CPI inflation rates, provides a sufficiently long time series of the whole cross-sectional distribution of inflation forecasts. We also compute disagreement about the mean from the cross-sectional distribution of inflation forecasts as a robustness check for our inflation disagreement results based on the CPI. Disagreement about the variance and skewness among professionals is computed from their individual responses for the probability of changes in the GDP deflator taken from the SPF database (series PRPGDP).[26] The MSC inflation forecasts, conducted at a monthly frequency, are available since January 1978 while the SPF inflation forecasts, conducted at a quarterly frequency, are available since September 1981. The GDP deflator forecasts for the current and next calendar year are also available since September 1981.

Yields. The U.S. Treasury only began issuing TIPS in 1997, so we merge the implied real yields in Chernov and Mueller (2012), which are available at quarterly frequency from $Q3$-1971 to $Q4$-2002, with real yields on Treasury Inflation Protected Securities (TIPS) to build a longer time series of real bond yields. The available real yield maturities are 2, 3, 5,

[25] See www.philadelphiafed.org/research-and-data for a detailed description of the Survey of Professional Forecasters, which is conducted by the Federal Reserve Bank of Philadelphia. The website www.sca.isr.umich.edu/ contains detailed information regarding the Michigan Surveys of Consumers.

[26] See Appendix B for more details on the data and computation of the two disagreement measures.

7, and 10 years.[27] Monthly nominal Fama-Bliss discount bond yields are from CRSP.[28] The Fama-Bliss discount bond file contains yields with 1 to 5 year maturities with data going back to 1952. Lastly, from the real and the nominal yield series, we compute the time series of real and nominal yield volatilities by estimating a GARCH$(1,1)$ model with an AR(1) mean equation. We use all available data in the GARCH estimation.

Cross-Sectional Consumption. We calculate monthly cross-sectional consumption growth volatility, starting from April 1984, from consumption growth rates of consumers using data from the Consumer Expenditure Survey (CEX) of the Bureau of Labor Statistics.[29] For further information regarding the CEX data and how to construct consumption growth rates of households from the raw data, see Malloy, Moskowitz, and Vissing-Jorgensen (2009) and the references therein.

Trading on Inflation Disagreement. We construct three measures for trading on inflation disagreement. First, we use the volatility of total Treasury volume scaled by outstanding Treasuries.[30] The trading volume data and the outstanding amount of Treasuries are available from the Securities Industry and Financial Markets Association (SIFMA) at a monthly frequency since January 2001.[31] To measure the volatility of trading in Treasuries, we estimate a GARCH$(1,1)$ model with a constant mean term. Second, we use the open interest in interest rate futures and scale it by the open interest of all financial futures to account for increased security trade over time. The open interest data for interest rate and financial futures are from the U.S. Commodity Futures Trading Commission (CFTC) at a monthly frequency since April 1986.[32] Third, we use de-trended log inflation swap notionals available at the monthly frequency since December 2005.[33] The monthly notional amounts correspond to averages of daily brokered inflation swap activity.

Inflation. We obtain quarterly and monthly CPI data from the Federal Reserve Eco-

[27]The real yield data are available at personal.lse.ac.uk/muellerp/RealYieldAOT5.xls. The TIPS data are available from Gürkaynak, Sack, and Wright (2010). For the 5, 7, and 10 year maturities, we use TIPS data from 2003 onwards. Given that ex ante real yields are not directly observable for most of the sample, but estimated using a term structure model, we show in the Internet Appendix that the results are robust to various alternative measures of ex ante real yields. The 4 year yield is not available in Chernov and Mueller (2012). For 2 and 3 year maturities, we interpolate the rates for 2003 with cubic splines.

[28]The Fama-Bliss discount bond file is available from wrds-web.wharton.upenn.edu/wrds.

[29]We thank Jing Yu for advising us on the use of the CEX data including how to compute the cross-sectional consumption growth volatility.

[30]We follow Grossman and Zhou (1996), Longstaff and Wang (2013), and Ehling and Heyerdahl-Larsen (2016) to capture the intensity of trading by using the volatility of turnover because turnover is not defined in a frictionless economy.

[31]The data are from SIFMA's website at this link: www.sifma.org.

[32]CFTC data are available from www.cftc.gov.

[33]See Fleming and Sporn (2013) for a description of the data. We thank Michael Fleming for sharing the aggregated inflation swap notional data with us.

nomic Data to compute inflation rates as logarithmic changes starting in January 1947. We estimate a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation using the whole sample, to obtain a time series of monthly and quarterly expected inflation and inflation volatility forecasts over multiple horizons.

Summary Statistics. We conclude this subsection with summary statistics of all variables in Tables 1 and 2. All data series end in June of 2014 or $Q2$-2014 except the CEX data (consumption is available until December 2012 and income until March 2012), Treasury volume (available until August 2013), open interest data (available until December 2013), and the inflation swap notionals (available until February 2012).

2.2 Nominal Yields

We show in this subsection that an increase in inflation disagreement raises nominal yields of all maturities when controlling for expected inflation. This is consistent with our theory which predicts that nominal yields increase with disagreement when risk aversion is greater than log utility and the consumption weighted-average inflation belief, in short the market view, does not change with disagreement.

Univariate regressions of nominal yields on inflation disagreement (not reported) lead to statistically and economically positive coefficients. Theoretically, this increase in nominal yields could be due to an increase in the market view about inflation rather than an increase in disagreement. To rule this out, we need to control for the market view about inflation which, unfortunately, is unobservable. Hence, we use expected inflation based on an ARMA(1,1) model as a proxy for the unobservable market view.[34]

Table 3 shows the slope coefficients, t-statistics, the R^2's, and the number of observations (N) for two multivariate regression models. For each maturity, we regress nominal yields on disagreement about inflation (DisInf) based on the SPF (columns 2 to 6) and the MSC (columns 7 to 11). To facilitate a comparison between the SPF and the MSC, we standardize the regression coefficients in all tables. To correct for serial correlation in error terms, we compute Newey-West corrected t-statistics with 12 lags in all regressions. We control for expected inflation (ExpInf) in regression model 1 as well as expected inflation and inflation volatility (SigInf) in regression model 2. The forecast horizons for ExpInf and SigInf corre-

[34] In Section 3, we calibrate a dynamic model, where investors disagree about the expected inflation rate, to disagreement, inflation, and yield data and show that using expected inflation, estimated as an ARMA(1,1) model, instead of the consumption share weighted-average belief does not cause a bias in the estimated coefficient and t-statistic of inflation disagreement.

Table 1: **Descriptive Statistics of Real and Nominal Yields and their Volatilities.** The table reports mean, median, standard deviation (Std), and number of observations (N) of percentage real and nominal yields and real and nominal yield volatilities. Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010). Nominal yields at monthly and quarterly frequency are from Fama-Bliss. Yield volatilities are computed by estimating a GARCH(1, 1) model with an AR(1) mean equation. Real yield sample: Q3-1981 to Q2-2014. Nominal yield sample: January 1978 to June 2014.

Maturity	2y	3y	5y	7y	10y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Quarterly Real Yields					Quarterly Nominal Yields					Monthly Nominal Yields				
Mean	1.927	2.027	2.255	2.415	2.586	4.851	5.160	5.421	5.647	5.809	5.412	5.672	5.884	6.080	6.220
Median	2.370	2.395	2.430	2.533	2.628	5.012	5.080	5.332	5.615	5.585	5.370	5.560	5.788	5.922	5.972
STD	1.976	1.836	1.594	1.433	1.247	3.390	3.424	3.380	3.311	3.219	3.690	3.631	3.528	3.432	3.328
N	132	132	132	132	132	132	132	132	132	132	438	438	438	438	438
	Quarterly Real Yield Volatilities					Quarterly Nominal Yield Volatilities					Monthly Nominal Yield Volatilities				
Mean	0.733	0.620	0.511	0.452	0.389	0.319	0.352	0.357	0.368	0.357	0.374	0.388	0.386	0.388	0.371
Median	0.639	0.555	0.468	0.423	0.368	0.258	0.313	0.335	0.339	0.329	0.278	0.322	0.340	0.342	0.326
STD	0.298	0.212	0.134	0.096	0.070	0.256	0.196	0.154	0.152	0.140	0.352	0.263	0.216	0.188	0.167

Table 2: **Descriptive Statistics of the Mean, Volatility, and Disagreement of Inflation, CEX and Trading Data.** The table reports mean, median, standard deviation (Std), and number of observations (N) of monthly and quarterly expected inflation, monthly and quarterly inflation volatility, MSC and SPF based measures of inflation disagreements (DisInf), CEX cross-sectional consumption growth volatility (Cons Vol) and income growth volatility (Income Vol), volatility of treasury volume (Vol Volume), open interest ratio in interest rate futures (Open Interest Ratio), and the notionals of inflation swaps (Inf Swaps). The reported statistics of one year forecasts of expected inflation (ExpInf) and inflation volatility (SigInf) are estimated using a GARCH$(1, 1)$ model with an ARMA$(1, 1)$ mean equation.

Forecast Horizon	1y	2y	3y	4y	5y	7y	10y	1y	2y	3y	4y	5y
	Quarterly Expected Inflation							Monthly Expected Inflation				
Mean	3.106	3.192	3.259	3.313	3.355	3.417	3.475	3.628	3.603	3.584	3.568	3.555
Median	3.100	3.186	3.254	3.308	3.351	3.414	3.473	3.197	3.239	3.272	3.299	3.320
STD	1.133	0.948	0.803	0.688	0.597	0.464	0.339	2.047	1.730	1.479	1.277	1.115
N	132	132	132	132	132	132	132	438	438	438	438	438
	Quarterly Inflation Volatility							Monthly Inflation Volatility				
Mean	1.746	1.483	1.359	1.286	1.237	1.175	1.124	0.973	0.722	0.608	0.538	0.489
Median	1.446	1.271	1.191	1.145	1.115	1.079	1.050	0.874	0.661	0.565	0.505	0.463
STD	0.942	0.668	0.533	0.450	0.392	0.316	0.249	0.363	0.217	0.153	0.115	0.091
N	132	132	132	132	132	132	132	438	438	438	438	438
	Quarterly			Monthly								
	DisInf SPF	DisInf MSC		DisInf MSC	CEX Cons Vol	CEX Income Vol	Vol Volume	Open Interest Ratio	Inf Swaps			
Mean	0.660	5.192		5.537	0.367	0.898	0.036	0.688	0.000			
Median	0.564	4.900		5.200	0.366	0.902	0.023	0.712	0.020			
STD	0.339	1.584		1.947	0.022	0.178	0.023	0.103	0.496			
N	132	132		438	345	330	152	333	70			

spond to the yield maturity in each regression. Specifically, we control for inflation volatility to address concerns that inflation disagreement raises nominal yields because of its positive correlation with inflation volatility.[35] Nominal yields are available at the monthly frequency and, thus, we use monthly data starting in January 1978 for the MSC. Hence, the sample size using the MSC is 438 and it is 132 when using the SPF.

Table 3: **Inflation Disagreement and Nominal Yields.** The table reports results from OLS regressions of nominal yields on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: Q3-1981 to Q2-2014 and January 1978 to June 2014.

	Survey of Professional Forecasters					Surveys of Consumers				
Maturity	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
DisInf	0.354	0.356	0.363	0.364	0.377	0.470	0.513	0.548	0.571	0.594
t-stat	3.63	3.60	3.65	3.74	3.88	4.11	4.39	4.61	4.80	5.05
ExpInf	0.459	0.449	0.437	0.435	0.424	0.356	0.298	0.249	0.219	0.196
t-stat	4.36	4.37	4.26	4.26	4.19	3.49	2.73	2.16	1.81	1.62
adj. R^2	0.41	0.40	0.39	0.39	0.39	0.58	0.56	0.55	0.54	0.55
N	132	132	132	132	132	438	438	438	438	438
DisInf	0.364	0.374	0.381	0.384	0.397	0.488	0.542	0.582	0.613	0.636
t-stat	3.51	3.50	3.51	3.58	3.63	4.16	4.55	4.86	5.17	5.42
ExpInf	0.448	0.430	0.416	0.411	0.399	0.334	0.264	0.207	0.169	0.144
t-stat	3.21	3.07	2.94	2.89	2.83	3.15	2.41	1.84	1.46	1.24
SigInf	-0.024	-0.041	-0.044	-0.049	-0.050	-0.061	-0.091	-0.107	-0.126	-0.126
t-stat	-0.24	-0.40	-0.42	-0.46	-0.47	-0.86	-1.31	-1.55	-1.88	-1.97
adj. R^2	0.40	0.39	0.39	0.39	0.39	0.58	0.57	0.56	0.56	0.56
N	132	132	132	132	132	438	438	438	438	438

The coefficients for disagreement are positive as well as economically and statistically significant for the SPF and MSC at all maturities, as shown in the top panel of Table 3. An increase in disagreement by one standard deviation for the SPF (0.339%) and the MSC (1.947%) raises the one year nominal yield by 35.4% and 47.0% of its standard deviation (3.390% and 3.690%, respectively). The economic significance of inflation disagreement is large and comparable to that of expected inflation, which is 45.9% and 35.6%, respectively across the two surveys. The results are similar for other maturities. The bottom panel of Table 3 shows that the coefficient estimates for disagreement remain positive and statistically significant when we control for the mean and volatility of inflation. All coefficient estimates

[35] We also normalize inflation disagreement by inflation volatility and show that inflation disagreement remains economically and statistically significant. For a detailed discussion of this measure, see Section 3.

for inflation volatility are negative and insignificant, except for maturities 4 and 5 years in the MSC regression which are negative and significant at the 5% level.

2.3 Real Yields

We show that an increase in inflation disagreement raises real yields at all maturities. This is consistent with our theoretical prediction when $\gamma > 1$, that is, when agents are more risk averse than log utility. Moreover, it confirms the economic channel through which nominal yields increase. All results presented in this section are based on the real yield data from Chernov and Mueller (2012), but they are robust to other proxies.[36]

Table 4 shows the slope coefficients, t-statistics, the R^2's, and the number of observations (N) for a univariate and a multivariate regression model. For each maturity, we regress real yields on disagreement about inflation (DisInf) based on the SPF (columns 2 to 6) and the MSC (columns 7 to 11). To facilitate a comparison between the SPF and the MSC, we use the sample period $Q3$-1981 to $Q2$-2014 and standardize the regression coefficients in all tables. To correct for serial correlation in error terms, we compute Newey-West corrected t-statistics with 12 lags in all regressions. The top panel of Table 4 shows the univariate regression results. The coefficient estimates for disagreement are positive and statistically significant, at least at the 5% level, for the SPF and the MSC at all maturities. Inflation disagreement is economically significant for the SPF and the MSC. Specifically, an increase in disagreement by one standard deviation of the SPF (0.339%) and the MSC (1.584%) raises the two year real yield by 40.7% and 56.0% of its standard deviation (1.976%). The results are similar for other maturities.

Inflation disagreement may be significant in a univariate regression because it correlates with other variables that impact real yields. For instance, empirical evidence such as in Christiano, Eichenbaum, and Evans (1999) shows that money is not neutral and, thus, expected inflation and inflation volatility can affect real yields. As both of these quantities are positively correlated with inflation disagreement in our data, we control for expected inflation (ExpInf) and inflation volatility (SigInf) in regression model 2. The bottom panel of Table 4 shows that the coefficient estimates for disagreement remain positive and statistically significant when we control for the mean and volatility of inflation. Expected inflation is positive and statistically significant for short maturities. For longer maturities,

[36] We subtract two different measures of expected inflation from nominal yields to compute two additional proxies for real yields and show in Table 1 of the Internet Appendix that inflation disagreement has an economically and statistically positive impact on real yields for all maturities.

the coefficients become insignificant for the MSC. Inflation volatility produces statistically insignificant coefficient estimates in all regressions.

Table 4: **Inflation Disagreement and Real Yields.** The table reports results from OLS regressions of real yields on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1, 1)$ model with an ARMA$(1, 1)$ mean equation over multiple horizons (T). Sample: $Q3$-1981 to $Q2$-2014.

	Survey of Professional Forecasters					Surveys of Consumers				
Maturity	2y	3y	5y	7y	10y	2y	3y	5y	7y	10y
DisInf	0.407	0.397	0.388	0.382	0.376	0.560	0.575	0.583	0.589	0.595
t-stat	3.48	3.33	3.23	3.18	3.12	3.04	3.18	3.29	3.39	3.50
adj. R^2	0.16	0.15	0.14	0.14	0.13	0.31	0.33	0.33	0.34	0.35
N	132	132	132	132	132	132	132	132	132	132
DisInf	0.290	0.285	0.281	0.280	0.280	0.452	0.472	0.487	0.501	0.515
t-stat	2.27	2.20	2.12	2.12	2.12	2.55	2.75	3.00	3.17	3.34
ExpInf	0.350	0.359	0.358	0.352	0.344	0.251	0.246	0.236	0.221	0.206
t-stat	2.19	2.17	2.03	1.95	1.88	1.98	1.98	1.87	1.77	1.64
SigInf	0.099	0.080	0.068	0.057	0.042	0.106	0.077	0.056	0.038	0.018
t-stat	0.71	0.57	0.48	0.39	0.29	1.04	0.80	0.62	0.43	0.20
adj. R^2	0.24	0.24	0.24	0.23	0.22	0.34	0.36	0.36	0.37	0.37
N	132	132	132	132	132	132	132	132	132	132

2.4 Real and Nominal Yield Volatilities

We now test whether real and nominal yield volatilities increase with inflation disagreement. Table 5 presents standardized coefficients and Newey-West adjusted t-statistics with 12 lags for the SPF in columns 2 to 6 and the MSC in columns 7 to 11. In both regressions, we control for the mean and volatility of inflation. Like the real and nominal yield levels, the coefficients for disagreement are positive and economically significant for the SPF and the MSC for all maturities. Table 5 shows that an increase in disagreement by one standard deviation for the SPF (0.339%) and the MSC (1.584%) raises the two year real yield volatility by 52.3% and 33.2% of its standard deviation (0.298%) and the one year nominal yield volatility by 59.7% and 47.4% of its standard deviation (0.256%). The results are similar for other maturities.

Table 5: **Inflation Disagreement and Real and Nominal Yield Volatilities.** The table reports results from OLS regressions of real and nominal yield volatilities on disagreement about inflation (DisInf), expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: $Q3$-1981 to $Q2$-2014 and January 1978 to June 2014.

	Survey of Professional Forecasters					Surveys of Consumers				
	Real Yield Volatilities									
Maturity	2y	3y	5y	7y	10y	2y	3y	5y	7y	10y
DisInf	0.523	0.560	0.624	0.700	0.749	0.332	0.387	0.420	0.447	0.471
t-stat	8.13	8.76	8.61	9.32	9.52	1.97	2.16	2.15	2.04	1.97
ExpInf	0.018	0.065	0.081	0.055	0.025	0.074	0.110	0.137	0.129	0.108
t-stat	0.20	0.75	0.94	0.60	0.25	0.61	0.96	1.20	1.02	0.80
SigInf	0.238	0.228	0.183	0.114	0.016	0.391	0.380	0.351	0.305	0.219
t-stat	2.17	2.08	1.84	1.40	0.22	2.87	2.87	2.66	2.19	1.51
adj. R^2	0.40	0.44	0.50	0.56	0.57	0.28	0.32	0.34	0.33	0.31
N	132	132	132	132	132	132	132	132	132	132
	Nominal Yield Volatilities									
Maturity	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
DisInf	0.597	0.606	0.567	0.656	0.644	0.474	0.464	0.442	0.501	0.511
t-stat	5.24	5.20	5.94	6.92	8.13	4.40	4.03	4.01	3.67	3.65
ExpInf	0.287	0.265	0.260	0.204	0.205	0.287	0.261	0.283	0.170	0.126
t-stat	2.90	2.74	2.44	2.25	2.31	1.60	1.44	1.50	0.96	0.68
SigInf	0.129	0.116	0.113	0.088	0.063	0.174	0.174	0.150	0.153	0.114
t-stat	1.37	1.21	1.15	1.09	0.81	2.45	2.26	2.08	1.99	1.45
adj. R^2	0.55	0.54	0.48	0.56	0.53	0.52	0.47	0.46	0.42	0.38
N	132	132	132	132	132	438	438	438	438	438

2.5 Economic Channel

Testing for the economic channel through which disagreement affects yields, we find that inflation disagreement raises the cross-sectional consumption growth volatility. The top panel of Table 6 shows two regression specifications (columns 2 to 3). In the first specification, we regress the CEX cross-sectional consumption growth volatility on the MSC inflation disagreement and time-dummies that control for changes in the definition of food consumption and for missing data at the beginning of 1986 and 1996 due to changes in the household identification numbers. The second specification contains the CEX cross-sectional income growth volatility as a control. The coefficient estimates on inflation disagreement in both regressions are positive with t-statistics of 2.22 and 2.89, respectively. Adding expected inflation and the volatility of inflation as additional explanatory variables into both regressions, shown in the bottom panel of Table 6 (columns 2 to 3), produces slightly lower coefficient estimates with t-statistics of 1.94 and 2.29. In the regressions shown in Table 6, we lag DisInf by two months. We motivate lagging DisInf given the quarterly frequency of the CEX interviews for a household. Even if the survey participants adjust consumption contemporaneously with inflation beliefs, current innovations in consumption due to DisInf are reflected in the CEX the earliest within the same month and the latest with a two month lag.

To provide further evidence for our economic channel, we consider three different classes of securities for which we expect increased trading when inflation disagreement is higher. First, inflation disagreement increases trading in nominal Treasury bonds. Column 4 in Table 6 shows a statistically positive relation between the MSC inflation disagreement and trading in Treasuries measured by the volatility of total Treasury volume scaled by outstanding Treasuries. The regressions differ in that in the bottom regression we add in ExpInf and SigInf as controls. The univariate regression produce a t-statistic of 2.33, while the multivariate regression produces a t-statistic of 3.78.

Second, inflation disagreement increases trading in interest rate futures. We use open interest in interest rate futures scaled by open interest in financial futures and present the evidence for this trading channel in column 5 of Table 6. The t-statistics for the regression coefficients on the MSC inflation disagreement are 2.60 (univariate) and 2.99 (multivariate using ExpInf and SigInf), respectively.

Third, inflation disagreement raises trading in inflation swaps. We measure inflation swap trading by detrending aggregated inflation notionals in both regressions. The univariate regression of inflation swap trading on the MSC DisInf produces a t-statistics of 4.35. The multivariate regression, shown in the bottom panel of Table 6, does not yield a statistically significant coefficient estimate, which is likely caused by multicollinearity.[37]

Table 6: **Cross-Sectional Consumption Growth Volatility and Trading.** The table reports OLS regression results. Dependent variables are cross-sectional consumption growth volatility, volatility of U.S. government bond trading volume, open interest of interest rate futures scaled by open interest in financial futures, and detrended inflation swap notional amounts. Explanatory variables are disagreement about inflation (DisInf), expected inflation (ExpInf), the volatility of inflation (SigInf), and CEX cross-sectional income growth volatility (SigInc). The CEX based regression contains a time-dummy and DisInf, ExpInf, and SigInf are lagged by two months. The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation. Samples: April 1984 - December 2012, January 2001 - August 2013, April 1986 - December 2013, May 2005 - February 2012.

	CEX Consumption Volatility I	CEX Consumption Volatility II	Volatility of Volume	Open Interest Ratio	Inflation Swaps
DisInf	0.162	0.146	0.332	0.314	0.265
t-stat	2.22	2.89	2.33	2.60	4.35
SigInc		0.303			
t-stat		4.31			
adj. R^2	0.37	0.49	0.10	0.10	0.06
N	345	330	151	333	70
DisInf	0.145	0.127	0.549	0.282	0.153
t-stat	1.94	2.29	3.78	2.99	1.34
ExpInf	0.036	0.068	-0.356	0.080	0.080
t-stat	0.43	1.06	-2.45	0.66	0.67
SigInf	-0.159	-0.069	-0.577	-0.402	0.228
t-stat	-2.24	-0.96	-3.15	-3.70	1.27
SigInc		0.281			
t-stat		3.92			
adj. R^2	0.40	0.50	0.31	0.28	0.05
N	345	330	151	333	70

2.6 Disagreement about the Variance and Skewness of Inflation

As the above results illustrate, expected inflation disagreement increases the level of real and nominal yields. Our theory in Section 1 is more general because real and nominal yields also increase when there is disagreement about other moments of inflation, not just the mean. To empirically test this prediction, we use the SPF to compute disagreement about the mean

[37] The regression produces a high F-statistic with an insignificant t-statistic for each variable.

(DisInfMean), which serves as a robustness check for the results of Subsections 2.2 and 2.3, disagreement about the variance (DisInfVar), and disagreement about the skewness (DisInfSkew) of the one year inflation rate based on the probability forecasts for the GDP deflator. We consider the GDP deflator instead of the CPI because probability forecasts based on the CPI are only available since the first quarter of 2007 whereas probability forecasts based on the GDP deflator are available since the third quarter of 1981.[38] The two measures of inflation are very similar, that is, the correlation between the cross-sectional average inflation rate based on CPI and the GDP deflator is 96.21%. Table 7 provides summary statistics for all three disagreement measures. Disagreement about expected inflation derived from the probability forecasts for the GDP deflator is slightly lower and less volatile than disagreement about expected inflation based on the CPI.[39] The three disagreement measures are positively correlated.

Table 7: **Summary Statistics - Disagreement about the Mean, Variance, and Skewness of Inflation.** The table reports summary statistics for disagreement about the mean (DisInfMean), disagreement about the variance (DisInfVar), and disagreement about the skewness (DisInfSkew) of inflation in percent. The three disagreement measures are calculated as the cross-sectional standard deviation of the individual mean, variance, and skewness of one-year inflation rates based on the probability forecasts for the GDP deflator provided by the Survey of Professional Forecasters. Samples: $Q3$-1981 to $Q2$-2014.

	Mean	Median	STD	Correlation		
				DisInfMean	DisInfVar	DisInfSkew
DisInfMean	0.5546	0.5174	0.1711	100	51.63	18.59
DisInfVar	0.0082	0.0071	0.0041		100	49.15
DisInfSkew	0.0034	0.0007	0.0075			100

Table 8 shows regression results of real and nominal yields on inflation disagreement. Panels 1, 2, and 3 of Table 8 show in univariate regressions that the coefficient of inflation disagreement about the mean, variance, and skewness is positive as well as economically and statistically significant. Disagreement about skewness shows the weakest relation and has the lowest explanatory power. This is not surprising given there is more noise in estimating skewness.[40] From Panel 4 in Table 8, we see that disagreement about skewness is no

[38]The survey respondents provide probability forecasters for the current and next calendar year which implies that the forecast horizon shrinks within both years. To keep the forecast horizon constant, we interpolate between the two probability forecasts (see Appendix B for details). The time series for the second probability forecast starts in the third quarter of 1981.

[39]There is less variation in the probability forecasts than in the mean forecasts for inflation. The cross-sectional mean, median, and standard deviation of one year inflation forecasts based on the GDP deflator are 0.6570%, 0.5943%, and 0.3126%, respectively, which is nevertheless very similar to the ones based on the CPI.

[40]A significant fraction of forecasters cluster their probability estimates in a few bins. The average number

longer significant when including all three disagreement measures as independent variables. Importantly, the economic and statistical significance of DisInfMean and DisInfVar is very similar. This remains the case, even when we control for expected inflation and the volatility of inflation, as shown in Panel 5, although the economic magnitudes are slightly lower.

2.7 Robustness

We conduct several robustness checks of our empirical results that we summarize in this section. Due to space constraints, we report the results in the Internet Appendix (IA).

To address the concern that the real yields data of Chernov and Mueller (2012) are measured with error that may correlate with inflation disagreement, we show that our results remain robust when we consider two alternative proxies for real yields constructed by subtracting two different measures of expected inflation from nominal yields. We consider an ARMA(1,1) expected inflation estimate in IA Table 1 and a VAR expected inflation estimate in IA Table 2. Specifically, expected inflation in IA Table 2 is predicted by regressing future inflation over the horizon of each bond on current inflation and yields with maturities ranging from one to five years.

The advantage of using the nominal zero-coupon yields data extracted from U.S. Treasury security prices by the method of Fama and Bliss (1987) in the main text is that yields are not computed through a fitted function which smooths across maturities. However, the disadvantage of the Fama and Bliss (1987) data are that the maturities only range until year five. Hence, we consider zero-coupon bond yields ranging from 1 year to 15 years extracted from U.S.[41] Treasury security prices by the method of Gürkaynak, Sack, and Wright (2007). The SPF based regressions are in IA Table 3 and the MSC based regressions are in IA Table 4.

Tables 5 and 6 in the Internet Appendix show that all our results are robust if we consider the cross-sectional variance and the interquartile range of individual forecasters as measures of disagreement, instead of the cross-sectional standard deviation. We also scale our disagreement measure by inflation volatility, to address the concern that in times when inflation volatility and disagreement is high (low), the risk-return trade-off for trading on inflation beliefs is low (high), and linearly controlling for inflation volatility (as done in our main regression specifications) may not be enough. Indeed, IA Tables 7, 8, and 9 confirm that our results are robust when scaling inflation disagreement by inflation volatility. In addition,

of bins is 4.004 with a standard deviation of 1.842. The median number of buckets is 4.

[41] Maturities beyond 15 years are not available before November 1985.

Table 8: **Real and Nominal Yield Levels and Disagreement about the Mean, Variance, and Skewness of Inflation.** The table reports results from OLS regressions of real and nominal yields on disagreement about the mean (DisInfMean), disagreement about the variance (DisInfVar), and disagreement about the skewness (DisInfSkew) of inflation. The three disagreement measures are calculated as the cross-sectional standard deviation of the individual mean, variance, and skewness of one year inflation rates based on the probability forecasts for the GDP deflator provided by the Survey of Professional Forecasters. The first three panels show univariate regression results of real and nominal yields onto each disagreement measure. In Panel 4, all three disagreements are included. Panel 5 also controls for expected inflation (ExpInf), and the volatility of inflation (SigInf). The t-statistics (t-stat) are Newey-West corrected with 12 lags. Regression coefficients are standardized. ExpInf and SigInf are predicted by a GARCH$(1,1)$ model with an ARMA$(1,1)$ mean equation over multiple horizons (T). Samples: $Q3$-1981 to $Q2$-2014.

	Real Yields					Nominal Yields				
Maturity	2y	3y	5y	7y	10y	1y	2y	3y	4y	5y
DisInfMean	0.44	0.44	0.45	0.46	0.46	0.47	0.47	0.48	0.49	0.50
t-stat	2.88	2.86	2.83	2.83	2.85	3.19	3.23	3.31	3.41	3.50
adj. R^2	0.18	0.19	0.20	0.20	0.21	0.21	0.22	0.23	0.23	0.24
DisInfVar	0.45	0.45	0.46	0.45	0.45	0.53	0.53	0.53	0.53	0.53
t-stat	3.06	3.04	2.96	2.93	2.91	4.29	4.19	4.16	4.17	4.18
adj. R^2	0.19	0.20	0.20	0.20	0.20	0.28	0.27	0.27	0.27	0.28
DisInfSkew	0.18	0.19	0.20	0.21	0.21	0.24	0.24	0.24	0.24	0.24
t-stat	2.31	2.33	2.31	2.31	2.32	3.07	3.08	3.05	3.03	3.06
adj. R^2	0.03	0.03	0.03	0.04	0.04	0.05	0.05	0.05	0.05	0.05
DisInfMean	0.28	0.29	0.30	0.31	0.31	0.26	0.27	0.29	0.30	0.30
t-stat	2.44	2.43	2.45	2.49	2.53	2.20	2.31	2.45	2.55	2.66
DisInfVar	0.32	0.31	0.30	0.29	0.29	0.40	0.39	0.38	0.37	0.38
t-stat	2.56	2.56	2.46	2.39	2.33	3.70	3.52	3.45	3.44	3.39
DisInfSkew	-0.03	-0.02	-0.00	0.01	0.01	-0.01	-0.00	-0.00	-0.00	-0.00
t-stat	-0.29	-0.21	-0.01	0.06	0.13	-0.09	-0.03	-0.00	-0.00	-0.02
adj. R^2	0.24	0.25	0.26	0.26	0.26	0.32	0.32	0.32	0.33	0.34
DisInfMean	0.22	0.22	0.24	0.25	0.26	0.19	0.20	0.22	0.23	0.24
t-stat	2.27	2.28	2.34	2.40	2.46	2.19	2.33	2.49	2.58	2.71
DisInfVar	0.24	0.24	0.23	0.23	0.23	0.32	0.31	0.31	0.30	0.31
t-stat	2.18	2.21	2.17	2.14	2.11	3.37	3.27	3.24	3.26	3.23
DisInfSkew	-0.03	-0.02	-0.01	-0.00	0.00	-0.03	-0.03	-0.03	-0.03	-0.03
t-stat	-0.27	-0.24	-0.08	-0.03	0.02	-0.37	-0.31	-0.28	-0.28	-0.28
ExpInf	0.28	0.28	0.28	0.26	0.25	0.39	0.37	0.36	0.35	0.33
t-stat	2.04	2.08	2.00	1.93	1.84	2.70	2.59	2.46	2.37	2.33
SigInf	0.12	0.09	0.07	0.06	0.04	0.02	0.00	-0.00	-0.01	-0.01
t-stat	1.13	0.94	0.80	0.65	0.46	0.17	0.00	-0.05	-0.13	-0.16
adj. R^2	0.29	0.29	0.30	0.30	0.30	0.44	0.43	0.43	0.43	0.43

in IA Table 10 we construct the first principal component from the SPF and the MSC inflation disagreement to show that our results are robust to this alternative disagreement measure.

We also address the concern that other forms of disagreement, that are correlated with inflation disagreement, may drive all our results. Specifically, IA Tables 11 and 12 show that inflation disagreement still has an economically and statistically positive impact on the level and volatility of yields when controlling for disagreement about real GDP growth based on the SPF. Disagreement about real GDP is statistically significant for the real and nominal yields levels and nominal yield volatility regressions, but insignificant for the real yield volatility regressions. IA Table 13 in the Internet Appendix shows that our results are robust to controlling for disagreement about earnings among analysts. Disagreement among analysts has a negative, but insignificant relation with real and nominal yield levels.

There are several empirical studies that use disagreement to proxy for economic uncertainty and, thus, one might be concerned that it is economic uncertainty and not disagreement that drives our results. For example, Bloom (2009) and Wright (2011) use disagreement among forecasters to measure uncertainty. Therefore, to address a possible omitted variable problem in our main regression specifications, we consider five different measures of economic uncertainty: i) volatility of real consumption growth estimated by a GARCH(1,1) model (IA Table 14), ii) volatility of real GDP growth estimated by a GARCH(1,1) model (IA Table 15), iii) volatility of industrial production estimated by a GARCH(1,1) model (IA Table 16), iv) Jurado, Ludvigson, and Ng (2015) Uncertainty Measure (IA Table 17), and v) Baker, Bloom, and Davis (2015) Uncertainty Measure (IA Table 18). Inflation disagreement is still statistically and economically significant after controlling for each of the first four uncertainty measures. Baker, Bloom, and Davis (2015) use the SPF-based inflation disagreement to construct their uncertainty measure and, thus, it is not surprising that inflation disagreement is insignificant after controlling for it.

Finally, as interest rates may depend on the output gap in a New-Keynesian model or, more generally, the state of the economy, we show that our results are robust to controlling for the output gap as constructed in Cooper and Priestley (2009) (IA Table 19) and the Stock and Watson quarterly measure of the NBER business cycle indicator (IA Table 20).

3 Model-Based Quantitative Evidence

Based on our theoretical and empirical evidence, we present a dynamic model that fits moments of inflation, inflation disagreement, and real and nominal yields and implies plausible Sharpe ratios for inflation risk to quantitatively reproduce the impact of inflation disagreement on yield curves.

3.1 Model

The exogenous real aggregate output process C_t follows a geometric Brownian motion with dynamics given by

$$dC_t = \mu_C C_t \, dt + \sigma_C C_t \, dz_{C,t}, \qquad C_0 > 0, \tag{3.1}$$

where z_C represents a real shock. The dynamics of the price level Π_t and the unobservable expected inflation rate x_t are

$$d\Pi_t = x_t \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_{\Pi,t}, \qquad dx_t = \kappa \left(\bar{x} - x_t\right) \, dt + \sigma_x \, dz_{x,t}, \qquad \Pi_0 = 1, \tag{3.2}$$

where $z_{\Pi,t}$ represents a nominal shock. The three Brownian motions $z_{C,t}$, $z_{\Pi,t}$, and $z_{x,t}$ are uncorrelated.

To obtain zero disagreement in the steady-state and a tractable stochastic disagreement process, we assume that investors agree on the long run mean $\bar{x}$ and the speed of mean reversion κ, but differ in their beliefs about the volatility of expected inflation, σ_x.[42] The dynamics of the price level and the best estimator for expected inflation as perceived by investor i are given by (Liptser and Shiryaev (1974a,b)):

$$d\Pi_t = x_t^i \Pi_t \, dt + \sigma_\Pi \Pi_t \, dz_{\Pi,t}^i, \quad dx_t^i = \kappa \left(\bar{x} - x_t^i\right) \, dt + \hat{\sigma}_x^i \, dz_{\Pi,t}^i, \quad x_0^i \sim N\left(\mu_{\bar{x},0}^i, \sigma_{x_0^i}^2\right). \tag{3.3}$$

The volatility $\hat{\sigma}_x^i$ is a function of κ and σ_x^i. Investors observe the price level for a sufficiently long time so that the perceived volatility, $\hat{\sigma}_x^i$, has reached its steady state level.[43]

Investors' nominal innovation processes are linked through the disagreement process Δ_t,

[42] The disagreement process is deterministic if there is only disagreement about the long run mean and it is not Markov if there is disagreement about the speed of mean reversion.

[43] The steady state level is $\hat{\sigma}_x^i = \sigma_\Pi \left(\sqrt{\kappa^2 + \left(\frac{\sigma_x^i}{\sigma_\Pi}\right)^2} - \kappa\right)$. Note that the perceived volatility of expected inflation $\hat{\sigma}_x^i$ is lower than σ_x^i, due to updating.

which summarizes current disagreement about expected inflation. Specifically,

$$dz^2_{\Pi,t} \;=\; dz^1_{\Pi,t} - \Delta_t dt, \qquad \Delta_t = \frac{x^2_t - x^1_t}{\sigma_\Pi}. \tag{3.4}$$

The disagreement process Δ_t follows an Ornstein-Uhlenbeck process

$$d\Delta_t = -\beta\Delta_t dt + \sigma_\Delta dz^1_{\Pi,t}, \qquad \beta = \frac{\kappa\sigma_\Pi + \hat{\sigma}^2_x}{\sigma_\Pi}, \qquad \sigma_\Delta = \frac{\hat{\sigma}^2_x - \hat{\sigma}^1_x}{\sigma_\Pi}, \tag{3.5}$$

and the dynamics of the likelihood ratio λ_t are

$$d\lambda_t = \Delta_t \lambda_t dz^1_{\Pi,t}. \tag{3.6}$$

We determine the disagreement measure over the horizon $T - t$ in the next Proposition.

Proposition 2. *The disagreement measure is*

$$\mathcal{D}_{t,T} \equiv \mathcal{D}\left(\Delta^2_t, T-t\right) = \frac{\sigma^2_\Delta}{4\beta} + \frac{1}{4\beta\,(T-t)}\left(\Delta^2_t - \frac{\sigma^2_\Delta}{2\beta}\right)\left(1 - e^{-2\beta(T-t)}\right). \tag{3.7}$$

Disagreement is strictly increasing in Δ^2_t and converges to $\frac{1}{2}\Delta^2_t$ and $\frac{\sigma^2_\Delta}{4\beta}$ as T goes to t and infinity, respectively. Hence, the instantaneous disagreement measure is given by $\frac{1}{2}\Delta^2_t$ and the long-run disagreement measure equals $\frac{\sigma^2_\Delta}{4\beta}$. In Section 2, we measure disagreement as the standard deviation of expected inflation across investors, which in the model is $\frac{1}{2}\sigma_\Pi \frac{1}{\kappa}\left(1 - e^{-\kappa}\right) \mid \Delta_t \mid$. Therefore, the empirical disagreement measure is strictly increasing in $\mathcal{D}\left(\Delta(t)^2, T-t\right)$ for any maturity $T - t$.

Each investor solves the consumption-savings problem given in equation (1.1). We conclude the description of the model by specifying an external habit process which helps match asset pricing moments.[44] Specifically,

$$\log(H_t) = \log(H_0)e^{-\delta t} + \delta \int_0^t e^{-\delta(t-a)} \log(C_a)\, da, \qquad \delta > 0, \tag{3.8}$$

where δ describes the dependence of H_t on the history of aggregate output. Relative log output $\omega_t = \log(C_t/H_t)$, a state variable in the model, follows a mean reverting process

$$d\omega_t = \delta(\bar{\omega} - \omega_t)\, dt + \sigma_C\, dz_{C,t}, \qquad \bar{\omega} = (\mu_C - \sigma^2_C/2)/\delta. \tag{3.9}$$

[44]See Abel (1990), Abel (1999), Chan and Kogan (2002), and Ehling and Heyerdahl-Larsen (2016).

Equilibrium consumption allocations and state price densities are given in Proposition 1.

3.2 Real Yields

We provide closed-form solutions of real bond prices in the next proposition.[45]

Proposition 3. *The real bond price, when γ is an integer is*

$$B_{t,T} = \sum_{k=0}^{\gamma} w_t^k B_{t,T}^k. \tag{3.10}$$

The stochastic weights w_t^k sum up to one and are given by

$$w_t^k = \binom{\gamma}{k} \frac{\lambda_t^{\frac{k}{\gamma}}}{\left(1+\lambda_t^{\frac{1}{\gamma}}\right)^{\gamma}} = \binom{\gamma}{k} f(\lambda_t)^{\gamma-k}(1-f(\lambda_t)^k. \tag{3.11}$$

$B_{t,T}^k$ is an exponential quadratic function of the state vector $Y_{1,t} = (\Delta_t, \omega_t)$:

$$B_{t,T}^k = exp\left(\mathcal{A}_B^k(T-t) + \mathcal{B}_B^k(T-t)'Y_{1,t} + Y_{1,t}'\mathcal{C}_B^k(T-t)Y_{1,t}\right), \tag{3.12}$$

where the coefficients $\mathcal{A}_B^k(\cdot), \mathcal{B}_B^k(\cdot), \mathcal{C}_B^k(\cdot)$ are solutions to ordinary differential equations summarized in Section 3 of the Internet Appendix.

The bond price in equation (3.10) is a weighted average of artificial bond prices that belong to the class of quadratic Gaussian term structure models. To gain intuition, we inspect the real short rate r_t which is the limit of the bond yield as maturity T approaches t:

$$r_t = \underbrace{\rho + \gamma\mu_C - \frac{1}{2}\gamma(\gamma+1)\sigma_C^2}_{CRRA} - \underbrace{\delta(\gamma-1)\omega_t}_{Habit} + \underbrace{\left(1-\frac{1}{\gamma}\right) f(\lambda_t)(1-f(\lambda_t))\frac{1}{2}\Delta_t^2}_{Disagreement}. \tag{3.13}$$

We see from equation (3.13) that the real short rate is the real short rate in a CRRA preferences representative investor economy plus two additional terms. The additional terms account for habit preferences and inflation disagreement. The impact from inflation disagreement on the real yield curve depends on the consumption share $f(\lambda_t)$, risk aversion γ,

[45] Our solution method relies on a binomial expansion similar to the approach in Yan (2008), Dumas, Kurshev, and Uppal (2009), and Bhamra and Uppal (2014). Alternatively, the model can be solved by the generalized transform analysis proposed in Chen and Joslin (2012).

and the instantaneous disagreement measure $\frac{1}{2}\Delta_t^2$. The real short rate does not depend on disagreement if $\gamma = 1$ and is increasing in disagreement when $\gamma > 1$ (the opposite is true when $\gamma < 1$).

3.3 Nominal Yields

We provide closed-form solutions of the nominal price of a nominal bond in the next proposition.

Proposition 4. *The nominal bond price, when γ is an integer, is*

$$P_{t,T} = \sum_{k=0}^{\gamma} w_t^k P_{t,T}^k, \tag{3.14}$$

where w_t^k is given in equation (3.11). $P_{t,T}^k$ is an exponential quadratic function of the state vector $Y_t = (x_t^1, \Delta_t, \omega_t)$:

$$P_{t,T}^k = \exp\left(\mathcal{A}_P^k(T-t) + \mathcal{B}_P^k(T-t)'Y_t + Y_t'\mathcal{C}_P^k(T-t)Y_t\right), \tag{3.15}$$

where the coefficients $\mathcal{A}_P^k(\cdot), \mathcal{B}_P^k(\cdot), \mathcal{C}_P^k(\cdot)$ are solutions to ordinary differential equations summarized in Section 3 of the Internet Appendix.

Similarly to the real bond price, the nominal bond price can be expressed as a weighed average of artificial bond prices that belong to the class of quadratic Gaussian term structure models. Taking the limit of the nominal bond yield as the maturity T approaches t, we obtain the nominal short rate

$$r_{P,t} = r_t + f_t x_t^1 + (1 - f_t)x_t^2 - \sigma_\Pi^2. \tag{3.16}$$

We see from equation (3.16) that the nominal short rate is the sum of the real short rate, the market view about expected inflation, and a Jensen's inequality term. The intuition for this is straightforward; when an investor has a larger consumption share, her view is more important in determining the price of the nominal bond. Hence, the market view replaces expected inflation in a standard economy with homogeneous beliefs.

The main channel through which inflation disagreement affects nominally interest rates becomes transparent through equation (3.16) of the nominal short rate. There is no inflation risk premium without disagreement and from the perspective of an outsider whose view coincides with the market view there is also no inflation risk premium with disagreement.

Therefore, an increase in inflation disagreement raises the real short rate and, consequently, also the nominal short rate.[46]

3.4 Calibration

We set the preference parameters (ρ, γ, δ) to match the level of nominal yields and the consumption parameters (μ_C, σ_C) are from Chan and Kogan (2002). The inflation parameters $(\bar{x}, \kappa, \sigma_x)$ and disagreement parameters (σ_x^1, σ_x^2) match the mean, standard deviation, and autocorrelation of the consensus belief and disagreement in the SPF. We set the belief of the econometrician such that $\hat{\sigma}_x$ equals $(\hat{\sigma}_x^1 + \hat{\sigma}_x^2)/2$. We use the SPF instead of the MSC because the SPF explicitly asks professionals about CPI growth and, thus, leads to lower disagreement. The last parameter σ_Π matches the volatility of inflation. Table 9 reports the parameters.

Table 9: **Parameters**

Parameter	Description	Value
Investors		
ρ	Time preference parameter	0.006
γ	Common risk aversion	7
δ	Habit parameter	0.050
f_0	Initial consumption allocation	0.5
Consumption		
μ_C	Expected consumption growth	0.0172
σ_C	Volatility of consumption growth	0.0332
Inflation		
σ_Π	Inflation volatility	0.02
$\bar{x}$	Long run mean of expected inflation	0.0317
κ	Mean reversion of expected inflation	0.19
$\hat{\sigma}_x$	Volatility of expected inflation	0.01
Disagreement		
$\hat{\sigma}_x^1$	Estimated volatility of expected inflation investor 1	0.0044
$\hat{\sigma}_x^2$	Estimated volatility of expected inflation investor 2	0.0156

To analyze the quantitative implications of the model, we generate 10,000 sample paths of 50 years of data by simulating from the model under the belief of the econometrician (σ_x) instead of the belief of one of the investors (σ_x^1 or σ_x^2). All statistics are based on averages

[46]Both investors' inflation views differ from the market view and, thus, they perceive positive inflation risk premiums on their investments.

across the 10,000 sample paths.[47] Table 10 shows the mean, volatility, and autocorrelation of the consensus forecast in the first panel and disagreement in the second panel. We compute the mean and volatility of expected inflation across investors to determine the consensus belief and disagreement. The model matches the mean, volatility, and to a lesser extent the autocorrelation of the consensus belief and disagreement. Table 11 reports the mean, standard deviation, and autocorrelation of real and nominal yields in the model and in the data. The model matches the level and volatility of real and nominal yields. The persistence of nominal yields in the model is lower than in the data, that is, the average autocorrelation across maturities is 0.65 in the model and 0.89 in the data.[48]

Table 10: **Disagreement about Inflation in the Model and the Data.** The table reports the mean, volatility, and annual autocorrelation for the consensus belief and inflation disagreement. We compute the mean and volatility of expected inflation across investors to determine the consensus belief and disagreement. The SPF statistics are based on the Survey of Professional Forecasters available at the quarterly frequency from $Q3-1981$ to $Q2-2014$. Model statistics are based on averages across 10,000 sample paths of 50 years of simulated data under the belief of the econometrician.

	Model	SPF
Consensus Belief		
Average	0.032	0.031
Volatility	0.013	0.012
Autocorrelation	0.703	0.683
Disagreement		
Average	0.005	0.007
Volatility	0.004	0.003
Autocorrelation	0.168	0.190

[47]This version of our model, as most continuous-time heterogeneous belief models, is not stationary and, thus, we cannot compute unconditional moments.

[48]The mean, volatility, and Sharpe ratio of the market portfolio defined as a claim to aggregate output are 3.8%, 16.4%, and 0.23, respectively.

Table 11: **Yields in the Model and the Data.** The table reports summary statistics for real and nominal yields. Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010) for the period $Q3$-1981 to $Q2$-2014. Monthly nominal Fama-Bliss discount bond yields are from CRSP for the period January 1978 to June 2014. Model statistics are based on averages across 10,000 sample paths of 50 years of simulated real and nominal yields and their volatilities under the belief of the econometrician.

	Real Yields		Nominal Yields	
Maturity	Model	Data	Model	Data
Average				
1	0.021		0.052	0.049
2	0.021	0.019	0.053	0.052
3	0.022	0.020	0.053	0.054
4	0.023		0.054	0.056
5	0.024	0.023	0.055	0.058
Volatility				
1	0.028		0.029	0.034
2	0.023	0.020	0.025	0.034
3	0.021	0.018	0.022	0.034
4	0.019		0.021	0.033
5	0.019	0.016	0.020	0.032
Autocorrelation				
1	0.45		0.47	0.75
2	0.59	0.66	0.60	0.76
3	0.68	0.70	0.68	0.77
4	0.73		0.73	0.76
5	0.76	0.73	0.76	0.78

3.5 Quantitative Effects of Inflation Disagreement

Figure 6 shows real and nominal yields with maturities ranging from 1 to 5 years for two realizations of current disagreement Δ. In the two plots, the black solid line corresponds to the steady state level of Δ, which is 0, and the blue dashed line corresponds to a one standard deviation increase in Δ, which is 0.5143. The plots show that inflation disagreement has an economically significant impact on real and nominal yields. The economic magnitudes are comparable to the data. Specifically, an increase in disagreement by one standard deviation raises the two-year real yield by 0.94% and the one-year nominal yield by 1.43%. The effects in the data are $0.407 \times 1.976 = 0.80\%$ for the two-year real yield and $0.354 \times 3.124 = 1.11\%$ for the one-year nominal yield. The economic significance for longer maturities is lower in the model than in the data as disagreement is less persistent in the model.

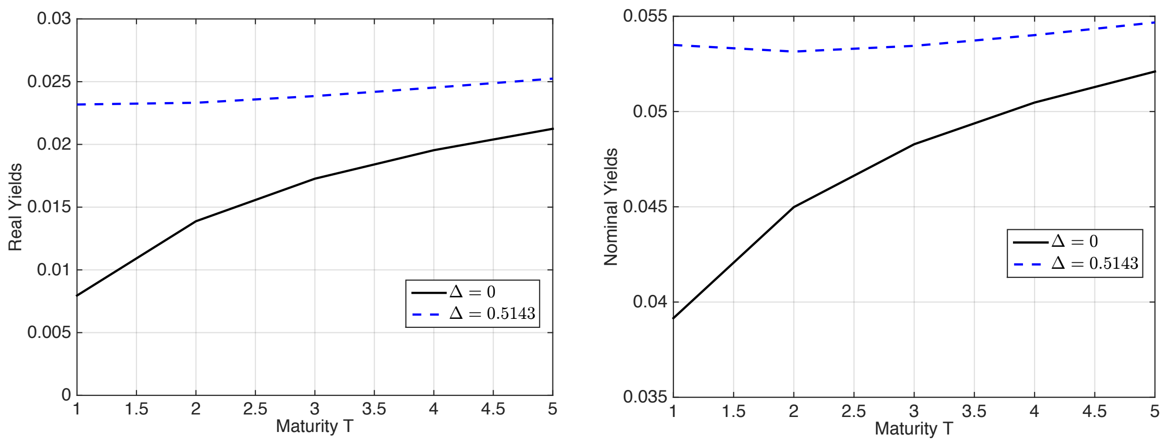

Figure 6: Real and Nominal Yields

The left plot shows real yields and the right plot shows nominal yields as function of time to maturity for two realizations of current disagreement Δ. The black solid line corresponds to the steady state level of Δ and the blue dashed line corresponds to a one standard deviation increase in Δ. Both plots show that an increase in inflation disagreement has an economically significant positive impact on real and nominal yields.

Table 12 shows regression results of real and nominal yields and their volatilities on disagreement about inflation and the econometrician's view about expected inflation. Coefficients and t-statistics for expected inflation are omitted to save space. As in the empirical analysis, the t-statistics are Newey-West corrected with 12 lags and coefficients are standardized in all four regressions. The coefficients, t-statistics, and R^2's for the real and nominal level and volatility regressions are similar to the data. In the second column of the nominal yield regression, we control for the market view about expected inflation instead of the econometrician's view. Using the econometrician's view instead of the market view about expected inflation does not lead to any noticeable differences and, hence, alleviates the concern that a measurement error may lead to biased coefficients and t-statistics in the empirical analysis.

3.6 Inflation Risk Premium and Sharpe Ratio

To inspect quantitatively the inflation risk premium and the corresponding Sharpe ratio, we specify a simple asset structure that dynamically completes the market.

Suppose investors can continuously trade an inflation-protected money market account with real price $B_{t,0}$, a nominal money market account with nominal price $P_{t,0}$, and a security, called a stock, with real price S_t and unit volatility that is locally perfectly correlated with real consumption growth. The dynamics of the inflation-protected money market account and stock in equilibrium are

$$dB_{t,0} = B_{t,0}\, r_t\, dt, \qquad B_{0,0} = 1, \qquad \text{and} \qquad dS_t = S_t \left((r_t + \theta_{C,t})\ dt + \ dz_{C,t} \right), \qquad S_0 = 1,$$

where $\theta_{C,t} = \gamma\, \sigma_C$ is the market price of risk for the real shock $z_{C,t}$. The dynamics of the real price of the nominal money market account, $p_{t,0} = P_{t,0}/\Pi_t$, in equilibrium, are

$$\begin{aligned} dp_{t,0} &= p_{t,0} \left(\left(r_{P,t} - x^i_t + \sigma^2_\Pi \right)\ dt - \sigma_\Pi\ dz^i_{\Pi,t} \right), \\ &= p_{t,0} \left(\left(r_t - \sigma_\Pi \theta^i_{\Pi,t} \right)\ dt - \sigma_\Pi\ dz^i_{\Pi,t} \right), \qquad p_{0,0} = 1, \qquad i = 1, 2, \end{aligned} \tag{3.17}$$

where $\theta^i_{\Pi,t}$ denotes the market price of risk of the perceived inflation shock $z^i_{\Pi,t}$. Specifically,

$$\theta^1_{\Pi,t} = (f_t - 1)\Delta_t \qquad \text{and} \qquad \theta^2_{\Pi,t} = f_t \Delta_t. \tag{3.18}$$

An increase in inflation is bad news for real asset prices and, thus, the market price of risk for the inflation shock has a different sign than the Sharpe ratio of the asset. The inflation risk premium and the Sharpe ratio for the nominal money market account perceived by investor i are

$$\mathrm{IRP}^i = -\sigma_\Pi \theta^i_{\Pi,t} \qquad \text{and} \qquad \mathrm{SR}^i = \frac{\mathrm{IRP}^i}{\sigma_\Pi} = -\theta^i_{\Pi,t}.$$

Hence, investors have opposing views on the real return of the nominal money market account due to their different views about expected inflation.

Table 12: **Inflation Disagreement Regressions.** The table reports results from OLS regressions of the level and volatility of real and nominal yields on disagreement about inflation and the econometrician's view about expected inflation. In the second column of the nominal yield regression, we control for the market view about expected inflation instead of the econometrician's view. Coefficients and t-statistics for expected inflation are omitted. We use the Survey of Professional Forecasters (SPF) to measure inflation disagreement ($Q3$-1981 to $Q2$-2014). Quarterly real yields are from Chernov and Mueller (2012) merged with TIPS yields from Gürkaynak, Sack, and Wright (2010). Monthly nominal Fama-Bliss discount bond yields are from CRSP. Real and nominal yield volatilities are computed by a GARCH$(1, 1)$ with an AR(1) mean equation. Model coefficients and standardized t-statistics are based on averages across 10,000 sample paths of 50 years of simulated real and nominal yields and their volatilities under the belief of the econometrician. The t-statistics (t-stat) are Newey-West corrected with 12 lags.

	Real Yields				Nominal Yields				
	Level		Volatility		Level			Volatility	
Maturity	Model	SPF	Model	SPF	Model		SPF	Model	SPF
1 year	0.635		0.574		0.599	0.599	0.354	0.540	0.659
t-stat	7.988		8.232		7.987	7.982	3.630	7.795	5.690
adj. R^2	0.485		0.392		0.533	0.533	0.406	0.407	0.552
2 year	0.490	0.333	0.531	0.642	0.456	0.455	0.406	0.478	0.670
t-stat	5.665	3.093	7.170	8.099	5.663	5.649	3.599	6.621	5.960
adj. R^2	0.340	0.241	0.349	0.382	0.413	0.412	0.398	0.382	0.558
3 year	0.385	0.320	0.480	0.672	0.356	0.356	0.363	0.413	0.640
t-stat	4.254	2.964	6.136	8.635	4.252	4.236	3.650	5.541	7.012
adj. R^2	0.257	0.243	0.301	0.429	0.341	0.340	0.390	0.359	0.512
4 year	0.316		0.424		0.291	0.290	0.364	0.353	0.704
t-stat	3.384		5.213		3.382	3.367	3.744	4.668	7.965
adj. R^2	0.210		0.253		0.296	0.295	0.389	0.338	0.578
5 year	0.265	0.310	0.368	0.720	0.246	0.246	0.377	0.303	0.680
t-stat	2.817	2.890	4.398	7.943	2.814	2.801	3.877	3.972	8.952
adj. R^2	0.184	0.239	0.210	0.504	0.267	0.266	0.390	0.318	0.550

We focus on the case where the first investor takes a long position in the nominal money market account because she perceives a positive inflation risk premium due to a lower expected inflation rate than the second investor, that is, $\Delta \geq 0$. The left plot of Figure 7 shows that the inflation risk premium and the Sharpe ratio perceived by the first investor are strictly increasing in disagreement Δ. The maximal Sharpe ratio and inflation risk premium when both investors share output equally ($f = 0.5$) and $\Delta = 0.5143$, which corresponds to a one standard deviation increase from the steady state of zero, are 0.2571 and 0.0051, respectively. As shown more generally in Proposition 1 of the Internet Appendix, the right plot of Figure 7 confirms that the inflation risk premium and the Sharpe ratio perceived by the first investor declines when her consumption share in the economy increases. When her consumption share is close to one, then prices reflect only her view about inflation and, thus, the inflation risk premium and the Sharpe ratio are close to zero. However, in this case the second investor perceives the highest Sharpe ratio and inflation risk premium in absolute terms because he is short the nominal money market account.

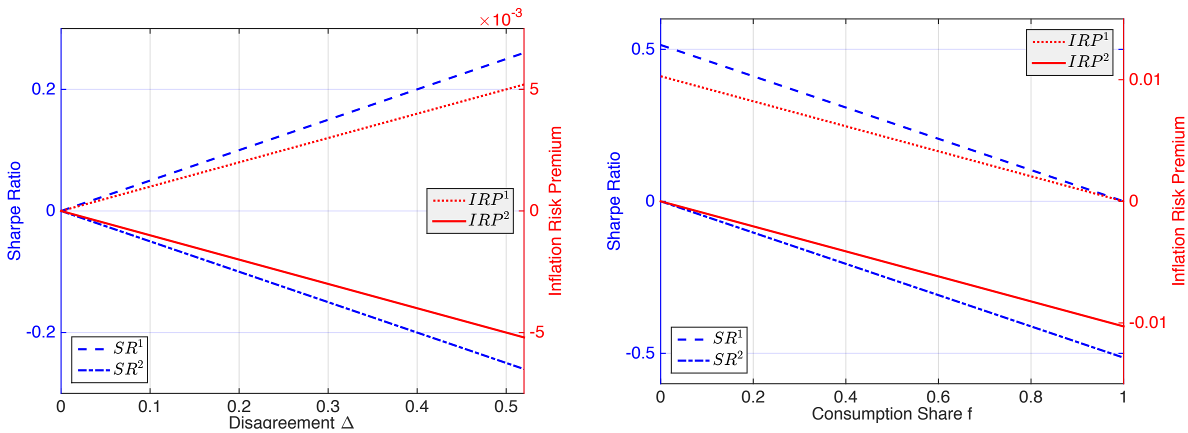

Figure 7: Sharpe Ratio and Inflation Risk Premium

The left plot shows the inflation risk premium and Sharpe ratio perceived by the first investor as strictly increasing functions of disagreement Δ. The second investor perceives a negative inflation risk premium and Sharpe ratio and thus is short the nominal money market account. The right plot shows that the inflation risk premium and Sharpe ratio perceived by the first investor goes down when her consumption share in the economy increases.

4 Concluding Remarks

Surveys of consumers and professionals show that there is disagreement about inflation. But does this disagreement affect asset prices or individual consumption? We consider a pure exchange economy with frictionless complete markets to answer this question theoretically. We show that disagreement about inflation has a strong impact on the cross-sectional consumption growth volatility as well as real and nominal yield curves. Intuitively, investors make different consumption-savings decisions based on their different beliefs about real returns on investments which raises the volatility of individual consumption and yields. Investors think that the high real returns on their investments will make them wealthier and, thus, interest rates have to rise for consumption markets to clear.

We find empirical support for our theoretical predictions using a survey of consumers and a survey of professionals. Specifically, real and nominal yields are higher and more volatile with inflation disagreement. The effects are economically and statistically significant. An inflation disagreement increase of one standard deviation raises real and nominal yields and their volatilities by at least 30% of their respective standard deviations. We provide empirical support for the economic channel through which disagreement effect asset prices by showing that there is more trade in nominal Treasuries, interest rate derivatives, and inflation swaps as well as higher cross-sectional consumption growth volatility when disagreement about inflation is high. Calibrating a dynamic model where investors disagree about the dynamics of expected inflation to disagreement, inflation, and yield data reproduces the economically and statistically significant impact of inflation disagreement on real and nominal yield curves.

We document that inflation disagreement raises individual consumption volatilities, real interest rates and their volatilities which seems to be an undesirable outcome for policymakers. Clearly, it is optimal for investors to trade on their inflation beliefs in our complete market economy. However, all investors cannot have correct beliefs and, thus, it is not clear whether trading on their beliefs is ex-post welfare improving. Recent studies such as Brunnermeier, Simsek, and Xiong (2014), Gilboa, Samuelson, and Schmeidler (2014), and Heyerdahl-Larsen and Walden (2015) show that policies that reduce disagreement or restrict trade on disagreement and, hence, avoid an increase in individual consumption volatilities, may be socially optimal in this case. Better understanding how central banks respond to inflation disagreement and potentially impact bond markets could be fruitful for future work.

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A Theoretical Results

Proof of Proposition 1. See Detemple and Murthy (1994) or Basak (2005) and the references therein. □

We focus in the main text on disagreement about the distribution of inflation (Assumption 1). In this appendix, we provide proofs for all statements in Section 1 under the more general Assumption 3, which allows for disagreement about the joint distribution of inflation, consumption, and the preference shock, not just disagreement about the distribution of inflation.

Assumption 3 (Inflation Disagreement)**.** *There is no disagreement about the distribution of consumption and the preference shock.*

Proof of Theorem 1. We prove Theorem 1 using Assumption 3 which implies Assumption 1. We split this proof into three parts

1. Real yields:

   Let ξ_T^0 denote the state price density when there is no disagreement. Specifically,

$$\xi_t^0 = e^{-\rho t} C_t^{-\gamma} H_t^{\gamma-1}.$$

   By Assumption 1, there is no disagreement about the distribution of output C_t and the habit H_t and, thus, the real price of a real bond when there is no disagreement and the representative investor has belief $\mathbb{P}^0$ is

$$B_{t,T}^0 = \mathbb{E}_t^0 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \right] = \mathbb{E}_t^2 \left[\frac{\xi_T^0}{\xi_t^0} \right].$$

   The real price of a real bond with disagreement is

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^1}{\xi_t^1} \right] = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(\frac{f(\lambda_T)}{f(\lambda_t)} \right)^{-\gamma} \right].$$

   We have that

$$\left(\frac{f(\lambda_T)}{f(\lambda_t)} \right)^{-\gamma} = \left(\frac{1 + (y\lambda_T)^{\frac{1}{\gamma}}}{1 + (y\lambda_t)^{\frac{1}{\gamma}}} \right)^{\gamma} = \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma},$$

   and, hence,

$$B_{t,T} = \mathbb{E}_t^1 \left[\frac{\xi_T^0}{\xi_t^0} \left(f_t + (1 - f_t) \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{1}{\gamma}} \right)^{\gamma} \right].$$

Suppose $\gamma = 1$. Then the bond price simplifies to

$$B_{t,T} = \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)\right)\right] = f_t\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\right] + (1-f_t)\mathbb{E}_t^1\left[\frac{\lambda_T}{\lambda_t}\frac{\xi_T^0}{\xi_t^0}\right]$$
$$= f_t\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\right] + (1-f_t)\mathbb{E}_t^2\left[\frac{\xi_T^0}{\xi_t^0}\right] = f_t B_{t,T}^0 + (1-f_t)B_{t,T}^0 = B_{t,T}^0.$$

This concludes the proof of the case $\gamma = 1$.

Consider the function $h(x) = x^{\frac{1}{\gamma}}$, which is strictly increasing and convex if $\gamma < 1$ and strictly concave if $\gamma > 1$. Suppose $\gamma > 1$ and, thus, $h(x)$ is strictly concave. The case of $\gamma < 1$ is similar and, thus, omitted.

The real price of a real bond with disagreement is

$$\begin{aligned}
B_{t,T} &= \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right] \\
&= \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\right]\mathbb{E}_t^1\left[\frac{\frac{\xi_T^0}{\xi_t^0}}{\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\right]}\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right] \\
&= B_{t,T}^0\,\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right],
\end{aligned}$$

where $\hat{\mathbb{E}}_t^1$ denotes the conditional mean using the bond price $B_{t,T}^0$ as numeraire. Specifically,

$$\frac{\zeta_T^1}{\zeta_t^1} \equiv \frac{d\hat{\mathbb{P}}^1}{d\mathbb{P}^1} = \frac{\xi_T^0}{\xi_t^0}\frac{1}{B_{t,T}^0}.$$

We have that

$$\hat{\mathbb{E}}_t^1\left[\frac{\lambda_T}{\lambda_t}\right] = \mathbb{E}_t^1\left[\frac{\lambda_T}{\lambda_t}\frac{\zeta_T^1}{\zeta_t^1}\right] = \mathbb{E}_t^2\left[\frac{\zeta_T^1}{\zeta_t^1}\right] = \frac{\mathbb{E}_t^2\left[\frac{\xi_T^0}{\xi_t^0}\right]}{B_{t,T}^0} = \frac{B_{t,T}^0}{B_{t,T}^0} = 1.$$

Strict concavity of $h(\cdot)$ leads to

$$f_t h(1) + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right) < h\left(f_t\cdot 1 + (1-f_t)\cdot\frac{\lambda_T}{\lambda_t}\right).$$

Hence,

$$B_{t,T} = B_{t,T}^0\,\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right] < B_{t,T}^0\,\hat{\mathbb{E}}_t^1\left[h\left(f_t\cdot 1 + (1-f_t)\cdot\frac{\lambda_T}{\lambda_t}\right)^{\gamma}\right]$$
$$= B_{t,T}^0\,\left(f_t + (1-f_t)\hat{\mathbb{E}}_t^1\left[\frac{\lambda_T}{\lambda_t}\right]\right) = B_{t,T}^0\,\left(f_t + (1-f_t)\right) = B_{t,T}^0.$$

2. *Real Yield Volatility*

If $\gamma = 1$, then real yields with disagreement are equal to real yields when there is no disagreement and, thus, the volatility of yields does not depend on disagreement.

Suppose $\gamma \neq 1$. The real price of a real bond with disagreement is

$$B_{t,T} = B^0_{t,T}\, \hat{\mathbb{E}}^1_t \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^\gamma \right],$$

where $\hat{\mathbb{E}}^1_t$ denotes the conditional mean using the real bond price without disagreement, $B^0_{t,T}$, as numeraire. Let $y^B_{t,T}$ denote the real yield when there is disagreement and $y^{B^0}_{t,T}$ the real yield when there is no disagreement. We have that

$$\begin{aligned} y^B_{t,T} &= -\frac{1}{T-t} \log\left(B_{t,T}\right) \\ &= -\frac{1}{T-t} \log\left(B^0_{t,T}\right) - \frac{1}{T-t} \log\left(\hat{\mathbb{E}}^1_t \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^\gamma \right] \right) \\ &= y^{B^0}_{t,T} - \frac{1}{T-t} \log\left(\hat{\mathbb{E}}^1_t \left[\left(f_t + (1 - f_t) h \left(\frac{\lambda_T}{\lambda_t} \right) \right)^\gamma \right] \right), \end{aligned} \tag{A.1}$$

and λ_t is independent of C_t and H_t and, hence,

$$\mathbb{V}^i \left[y^B_{t,T} \right] \geq \mathbb{V}^i \left[y^{B^0}_{t,T} \right], \qquad \forall i = 0, 1, 2,$$

with equality if the conditional expectation in equation (A.1) is constant.

$\square$

Corollary 1. *[Implications of Second-Order Stochastic Dominance]*

Consider the probability space $(\Omega, \mathcal{F})$ and the three strictly positive random variables $\tilde{x}, \tilde{y}$, and $\tilde{\varepsilon}$ with corresponding probability measures $\mathbb{P}^x$, $\mathbb{P}^y$, and $\mathbb{P}^\varepsilon$. Suppose that (i) $\tilde{y}$ and $\tilde{x}$ have unit mean, that is, $\mathbb{E}^y[\tilde{y}] = \mathbb{E}^x[\tilde{x}] = 1$, (ii) $\tilde{y}$ and $\tilde{x}\,\tilde{\varepsilon}$ are equal in distribution, that is, $\tilde{y} \overset{d}{=} \tilde{x}\,\tilde{\varepsilon}$, and (iii) $\tilde{x}$ and $\tilde{\varepsilon}$ are mean independent, that is, $\mathrm{E}^\varepsilon[\tilde{\varepsilon} \mid \tilde{x} = x] = \mathrm{E}^\varepsilon[\tilde{\varepsilon}] = 1$, $\forall x$. Then the following three statements hold:

1. $$\mathbb{E}^y\left[g\left(\tilde{y}\right)\right] \leq \mathbb{E}^x[g(\tilde{x})],$$
   for all concave functions g,
2. $$\mathbb{V}^y\left[\tilde{y}\right] \geq \mathbb{V}^x\left[\tilde{x}\right],$$
3. *and*
   $$\mathbb{E}^y\left[\left(\log\left(\tilde{y}\right)\right)^2\right] \geq \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right],$$
   if $\tilde{x}$ and $\tilde{\varepsilon}$ are independent.

Proof. We split the proof into three parts:

1. It follows from the definition of equality in distribution, mean independence, and Jensen's inequality that

$$\begin{aligned}\mathbb{E}^y\left[g\left(\tilde{y}\right)\right] &= \mathbb{E}^y\left[g\left(\tilde{x}\tilde{\varepsilon}\right)\right] = \mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[g\left(\tilde{x}\tilde{\varepsilon}\right) \mid \tilde{x}\right]\right] \leq \mathbb{E}^x\left[g\left(\mathbb{E}^\varepsilon\left[\tilde{x}\tilde{\varepsilon} \mid \tilde{x}\right]\right)\right] = \mathbb{E}^x\left[g\left(\tilde{x}\mathbb{E}^{\mathbb{P}}\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right)\right] \\ &= \mathbb{E}^x\left[g\left(\tilde{x}\right)\right].\end{aligned}$$

2. It follows from the definition of equality in distribution, mean independence, and Jensen's inequality that

$$\begin{aligned}\mathbb{V}^y\left[\tilde{y}\right] &= \mathbb{V}^y\left[\tilde{x}\,\tilde{\varepsilon}\right] = \mathbb{E}^y\left[\tilde{x}^2\tilde{\varepsilon}^2\right] - \left(\mathbb{E}^y\left[\tilde{x}\,\tilde{\varepsilon}\right]\right)^2 = \mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[\tilde{x}^2\tilde{\varepsilon}^2 \mid \tilde{x}\right]\right] - \left(\mathbb{E}^x\left[\mathbb{E}^\varepsilon\left[\tilde{x}\,\tilde{\varepsilon} \mid \tilde{x}\right]\right]\right)^2 \\ &= \mathbb{E}^x\left[\tilde{x}^2\mathbb{E}^\varepsilon\left[\tilde{\varepsilon}^2 \mid \tilde{x}\right]\right] - \left(\mathbb{E}^x\left[\tilde{x}\,\mathbb{E}^\varepsilon\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right]\right)^2 \geq \mathbb{E}^x\left[\tilde{x}^2\left(\mathbb{E}^\varepsilon\left[\tilde{\varepsilon} \mid \tilde{x}\right]\right)^2\right] - \left(\mathbb{E}^x\left[\tilde{x}\right]\right)^2 \\ &= \mathbb{E}^x\left[\tilde{x}^2\right] - \left(\mathbb{E}^x\left[\tilde{x}\right]\right)^2 = \mathbb{V}^x\left[\tilde{x}\right].\end{aligned}$$

3. Since $g(x) = \log(x)^2$ is convex for $0 < x < 1$ and concave for $x > 1$, we cannot apply the first result to show the third result. However, if $\tilde{x}$ and $\tilde{\varepsilon}$ are independent, then

$$\begin{aligned}\mathbb{E}^y\left[\left(\log\left(\tilde{y}\right)\right)^2\right] &= \mathbb{E}^y\left[\left(\log\left(\tilde{x}\,\tilde{\varepsilon}\right)\right)^2\right] = \mathbb{E}^y\left[\left(\log\left(\tilde{x}\right) + \log\left(\tilde{\varepsilon}\right)\right)^2\right] \\ &= \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right] + 2\mathbb{E}^y\left[\log\left(\tilde{x}\right)\log\left(\tilde{\varepsilon}\right)\right] + \mathbb{E}^\varepsilon\left[\left(\log\left(\tilde{\varepsilon}\right)\right)^2\right] \\ &= \mathbb{E}^x\left[\left(\log\left(\tilde{x}\right)\right)^2\right] + 2\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right]\mathbb{E}^\varepsilon\left[\log\left(\tilde{\varepsilon}\right)\right] + \mathbb{E}^\varepsilon\left[\left(\log\left(\tilde{\varepsilon}\right)\right)^2\right].\end{aligned}$$

   The first and third terms are non-negative and, thus, it remains to be shown that the second term is nonnegative. We know that $\tilde{x}$ and $\tilde{\varepsilon}$ have unit mean and, thus, the average of the log of both variables is nonpositive because by Jensen's inequality

$$\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right] \leq \log\left(\mathbb{E}^x\left[\tilde{x}\right]\right) = 0.$$

   Hence,

$$\mathbb{E}^x\left[\log\left(\tilde{x}\right)\right]\mathbb{E}^\varepsilon\left[\log\left(\tilde{\varepsilon}\right)\right] \geq 0,$$

which concludes the proof of the third statement. □

Definition 3. *[Second-Order Stochastic Dominance]*

Suppose the likelihood ratios η_u and λ_u are independent of C_u and H_u. Moreover, the distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$ conditional on $\mathcal{F}_{t_\eta}$ equals the distribution of $\frac{\lambda_T}{\lambda_t}\varepsilon$ conditional on $\mathcal{F}_t$, where ε denotes a strictly positive random variable with $\mathbb{E}^\varepsilon\left[\varepsilon \mid \lambda_T\right] = \mathbb{E}^\varepsilon\left[\varepsilon\right] = 1$ for all λ_T.

The concept of second-order stochastic dominance allows us to focus on one-dimensional decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$. The multiplicative decomposition of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$ nevertheless covers a large class of stochastic processes.[49]

[49]All results still follow if we consider additive mean independent and comonotone decompositions of the conditional distribution of $\frac{\eta_{T_\eta}}{\eta_{t_\eta}}$.

For instance, λ_t second-order stochastic dominates η_t in all three examples if $\Delta_\eta \geq \Delta$. Intuitively, one can think of η_{T_η} as a noisy version of λ_T.

Proof of Theorem 2. We split this proof into three parts

1. Disagreement:

   We need to show that if λ_t second order stochastically dominates η_t, then the belief structure $\mathcal{B}^\eta_{t_\eta,T_\eta}$ exhibits more disagreement than the belief structure $\mathcal{B}_{t,T}$, that is, $\mathcal{D}^\eta_{t_\eta,T_\eta} \geq \mathcal{D}_{t,T}$. Specifically,

$$\mathcal{D}_{t_\eta,t_\eta+\tau} = -\frac{1}{\tau}\mathbb{E}^{\eta,1}_{t_\eta}\left[\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] \geq -\frac{1}{\tau}\mathbb{E}^1_t\left[\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right] = \mathcal{D}_{t,t+\tau},$$

   which is equivalent to showing that

$$\mathbb{E}^{\eta,1}_{t_\eta}\left[\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] \leq \mathbb{E}^1_t\left[\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right]. \tag{A.2}$$

   The function $g(x) = \log(x)$ is concave and, thus, it follows from Corollary 1 that inequality (A.2) is satisfied if λ_t second order stochastically dominates η_t.

2. Real yields:

   Let ξ^0_T denote the state price density when there is no disagreement. Specifically,

$$\xi^0_t = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}.$$

   It follows from second-order stochastic dominance (Definition 3) that there is no disagreement about the distribution of output C_t and the habit H_t and, thus, the real price of a real bond when there is no disagreement and the representative investor has belief $\mathbb{P}^0$ is

$$B^0_{t,T} = \mathbb{E}^0_t\left[\frac{\xi^0_T}{\xi^0_t}\right] = \mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\right] = \mathbb{E}^2_t\left[\frac{\xi^0_T}{\xi^0_t}\right].$$

   The likelihood ratio λ_t is independent of ξ^0_t and, thus, the real price of a real bond with disagreement is

$$\begin{aligned}
B_{t,T} &= \mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\left(f(\lambda_t) + (1-f(\lambda_t))\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^\gamma\right] \\
&= \mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\right]\mathbb{E}^1_t\left[\left(f(\lambda_t) + (1-f(\lambda_t))\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^\gamma\right] \\
&= B^0_{t,T}\mathbb{E}^1_t\left[\left(f(\lambda_t) + (1-f(\lambda_t))\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^\gamma\right].
\end{aligned}$$

Similarly,

$$B^{\eta}_{t_\eta,T_\eta} = B^0_{t_\eta,T_\eta}\mathbb{E}^{\eta,1}_{t_\eta}\left[\left(f(\eta_{t_\eta}) + (1-f(\eta_{t_\eta}))\left(\frac{\eta_{T_\eta}}{\eta_{t_\eta}}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right].$$

We have that $\tau = T_\eta - t_\eta = T - t$ and, thus, $B^0_{t,t+\tau} = B^0_{t_\eta,t_\eta+\tau}$. Moreover, $f_t = f(\lambda_t) = f(\eta_{t_\eta})$ and, hence,

$$B_{t,t+\tau} = B^0_{t,t+\tau}\mathbb{E}^1_t\left[\left(f_t + (1-f_t)\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right],$$
$$B^{\eta}_{t_\eta,t_\eta+\tau} = B^0_{t,t+\tau}\mathbb{E}^{\eta,1}_{t_\eta}\left[\left(f_t + (1-f_t)\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right].$$

Suppose $\gamma = 1$. Then the bond prices simplify to

$$B_{t,t+\tau} = B^0_{t,t+\tau}\mathbb{E}^1_t\left[f_t + (1-f_t)\frac{\lambda_{t+\tau}}{\lambda_t}\right] = B^0_{t,t+\tau},$$
$$B^{\eta}_{t_\eta,t_\eta+\tau} = B^0_{t,t+\tau}\mathbb{E}^{\eta,1}_{t_\eta}\left[f_t + (1-f_t)\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right] = B^0_{t,t+\tau}.$$

This concludes the proof for the case when $\gamma = 1$.

Define the function $g(x) = \left(f + (1-f)x^{\frac{1}{\gamma}}\right)^{\gamma}$ which is strictly concave if $\gamma > 1$ and strictly convex if $\gamma < 1$. Suppose $\gamma > 1$ and, thus, $h(x)$ is strictly concave. The case of $\gamma < 1$ is similar and, thus, omitted. We need to show that $B^{\eta}_{t_\eta,t_\eta+\tau} < B_{t,t+\tau}$, which is equivalent to showing that

$$\mathbb{E}^{\eta,1}_{t_\eta}\left[g\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right] < \mathbb{E}^1_t\left[h\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right],$$

which follows directly from second-order stochastic dominance (Definition 3) and Corrolary 1.

3. *Consumption Growth Volatility*

   The cross-sectional variance of consumption growth from time t to T in both economies is

$$\sigma^2_{\mathrm{CS}}(\lambda_t,\lambda_{t+\tau}) = \frac{1}{4\gamma^2}\left(\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right)^2, \quad \text{and} \quad \sigma^2_{\mathrm{CS}}(\eta_{t_\eta},\eta_{t_\eta+\tau}) = \frac{1}{4\gamma^2}\left(\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right)^2.$$

   Hence, we need to show that

$$\mathbb{E}^{\eta,1}_{t_\eta}\left[\sigma^2_{\mathrm{CS}}(\eta_{t_\eta},\eta_{t_\eta+\tau})\right] \geq \mathbb{E}^1_t\left[\sigma^2_{\mathrm{CS}}(\lambda_t,\lambda_{t+\tau})\right],$$

which is equivalent to showing that

$$\mathbb{E}_{t_\eta}^{\eta,1}\left[\left(\log\left(\frac{\eta_{t_\eta+\tau}}{\eta_{t_\eta}}\right)\right)^2\right] \geq \mathbb{E}_t^1\left[\left(\log\left(\frac{\lambda_{t+\tau}}{\lambda_t}\right)\right)^2\right]. \tag{A.3}$$

If λ_t second-order stochastic dominates η_t (Definition 3) and if ε and $\lambda_{t+\tau}$ are independent, then inequality (A.3) follows from Corollary 1.

□

We generalize the assumptions for Theorem 3 of the main text to allow for (i) disagreement about the joint distribution of inflation, consumption, and the preference shock (Assumption 3). In this case, we require that the weighted average across each investor's inflation risk premium belief is fixed when inflation disagreement changes. Hence, we add restriction (A.5) to the definition of the market view. If inflation is independent of consumption and the preference shock (Assumption 2) or if there is only disagreement about expected inflation, then equation (A.5) is trivially satisfied. We also allow for disagreement about higher order moments of inflation and the joint distribution of inflation and real quantities and, hence, the beliefs $\mathbb{P}^1$ and $\mathbb{P}^2$ about the covariances in equation (A.5) do not have to be the same.

Definition 4 (Market View or Belief)**.** *Let $\mathbb{P}^0$ denotes the market belief that satisfies*

$$\mathbb{E}_t^0\left[\frac{\Pi_t}{\Pi_T}\right] = f(\lambda_t)\mathbb{E}_t^1\left[\frac{\Pi_t}{\Pi_T}\right] + (1-f(\lambda_t))\mathbb{E}_t^2\left[\frac{\Pi_t}{\Pi_T}\right], \tag{A.4}$$

$$\mathbb{C}ov_t^0\left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0}\right] = f(\lambda_t)\mathbb{C}ov_t^1\left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0}\right] + (1-f(\lambda_t))\mathbb{C}ov_t^2\left[\frac{\Pi_t}{\Pi_T}, \frac{\xi_T^0}{\xi_t^0}\right], \tag{A.5}$$

where $\xi_t^0 = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}$ is the state price density when there is no disagreement.

Theorem 4 imposes less restrictions on the likelihood ratio than Theorem 3 but requires the more general Definition 4 of the market view about expected inflation than Definition (2).

Theorem 4 (Nominal Yield)**.** *Fix the market view in Definition 4 and suppose Assumption 3 is satisfied, then*

1. *the break-even inflation rate and nominal yields do not depend on disagreement if $\gamma = 1$ and*

2. *nominal yields are higher with than without disagreement if $\gamma > 1$ (the opposite is true if $\gamma < 1$) even though the effects of inflation disagreement on the break-even inflation rate are ambiguous if $\gamma \neq 1$.*

Proof of Theorem 4. Let ξ_T^0 denote the state price density when there is no disagreement and the representative investor has belief $\mathbb{P}^0$. Specifically,

$$\xi_t^0 = e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}.$$

The nominal price of a nominal bond when there is no disagreement and the representative investor has belief $\mathbb{P}^i$ is

$$\bar{P}^i_{t,T} = \mathbb{E}^i_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right], \qquad i = 0, 1, 2.$$

The nominal price of a nominal bond with disagreement is

$$P_{t,T} = \mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right].$$

Suppose $\gamma = 1$. Then, the bond price simplifies to

$$\begin{aligned}P_{t,T} &= \mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)\right)\right] = f_t\mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}^1_t\left[\frac{\lambda_T}{\lambda_t}\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right]\\ &= f_t\mathbb{E}^1_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}^2_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right] = f_t\bar{P}^1_{t,T} + (1-f_t)\bar{P}^2_{t,T}.\end{aligned}$$

It remains to be shown that

$$f_t\bar{P}^1_{t,T} + (1-f_t)\bar{P}^2_{t,T} = \bar{P}^0_{t,T}.$$

We have for all beliefs indexed by $i = 0, 1, 2$ that

$$\bar{P}^i_{t,T} = \mathbb{E}^i_t\left[\frac{\xi^0_T}{\xi^0_t}\frac{\Pi_t}{\Pi_T}\right] = \mathbb{C}ov^i_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right] + \mathbb{E}^i_t\left[\frac{\xi^0_T}{\xi^0_t}\right]\mathbb{E}^i_t\left[\frac{\Pi_t}{\Pi_T}\right].$$

By Assumption 3, there is no disagreement about the marginal distribution of output and the habit and, hence,

$$B_{t,T} \equiv \mathbb{E}^i_t\left[\frac{\xi^0_T}{\xi^0_t}\right], \qquad \forall i = 0, 1, 2.$$

Therefore,

$$\bar{P}^1_{t,T} = \mathbb{C}ov^1_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}^1_t\left[\frac{\Pi_t}{\Pi_T}\right], \quad \text{and} \quad \bar{P}^2_{t,T} = \mathbb{C}ov^2_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}^2_t\left[\frac{\Pi_t}{\Pi_T}\right].$$

Multiplying the first equation with f_t and the second equation with $(1-f_t)$, adding them up, and imposing that the market view (see Definition 4) does not change with disagreement, leads to

$$\begin{aligned}P_{t,T} &= f_t\bar{P}^1_{t,T} + (1-f_t)\bar{P}^2_{t,T} = f_t\mathbb{C}ov^1_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{C}ov^2_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right]\\ &\quad + B_{t,T}\left(f_t\mathbb{E}^1_t\left[\frac{\Pi_t}{\Pi_T}\right] + (1-f_t)\mathbb{E}^2_t\left[\frac{\Pi_t}{\Pi_T}\right]\right) = \mathbb{C}ov^0_t\left[\frac{\xi^0_T}{\xi^0_t}, \frac{\Pi_t}{\Pi_T}\right] + B_{t,T}\mathbb{E}^0_t\left[\frac{\Pi_t}{\Pi_T}\right] = \bar{P}^0_{t,T}.\end{aligned}$$

This concludes the proof of the case $\gamma = 1$.

Consider the function $h(x) = x^{\frac{1}{\gamma}}$, which is strictly convex if $\gamma < 1$ and strictly concave if $\gamma > 1$. Suppose $\gamma > 1$. The case of $\gamma < 1$ is similar and, thus, omitted.

The nominal price of a nominal bond with disagreement is

$$\begin{aligned}
P_{t,T} &= \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\left(f_t + (1-f_t)\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{1}{\gamma}}\right)^{\gamma}\right] \\
&= \mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right]\mathbb{E}_t^1\left[\frac{\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}}{\mathbb{E}_t^1\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right]}\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right] \\
&= \bar{P}_{t,T}^1\,\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right],
\end{aligned}$$

where $\hat{\mathbb{E}}_t^1$ denotes the conditional mean using the bond price $\bar{P}_{t,T}^1$ as numeraire. Specifically,

$$\frac{\zeta_T^1}{\zeta_t^1} \equiv \frac{d\hat{\mathbb{P}}^1}{d\mathbb{P}^1} = \frac{\xi_T^0\Pi_T^{-1}}{\xi_t^0\Pi_t^{-1}}\frac{1}{\bar{P}_{t,T}^1}.$$

We have that

$$\hat{\mathbb{E}}_t^1\left[\frac{\lambda_T}{\lambda_t}\right] = \mathbb{E}_t^1\left[\frac{\lambda_T}{\lambda_t}\frac{\zeta_T^1}{\zeta_t^1}\right] = \mathbb{E}_t^2\left[\frac{\zeta_T^1}{\zeta_t^1}\right] = \frac{\mathbb{E}_t^2\left[\frac{\xi_T^0}{\xi_t^0}\frac{\Pi_t}{\Pi_T}\right]}{\bar{P}_{t,T}^1} = \frac{\bar{P}_{t,T}^2}{\bar{P}_{t,T}^1}.$$

Strict concavity of $h(\cdot)$ implies that

$$f_t h(1) + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right) < h\left(f_t\cdot 1 + (1-f_t)\cdot\frac{\lambda_T}{\lambda_t}\right).$$

Hence,

$$\begin{aligned}
P_{t,T} &= \bar{P}_{t,T}^1\,\hat{\mathbb{E}}_t^1\left[\left(f_t + (1-f_t)h\left(\frac{\lambda_T}{\lambda_t}\right)\right)^{\gamma}\right] < \bar{P}_{t,T}^1\,\hat{\mathbb{E}}_t^1\left[h\left(f_t\cdot 1 + (1-f_t)\cdot\frac{\lambda_T}{\lambda_t}\right)^{\gamma}\right] \\
&= \bar{P}_{t,T}^1\left(f_t + (1-f_t)\hat{\mathbb{E}}_t^1\left[\frac{\lambda_T}{\lambda_t}\right]\right) = \bar{P}_{t,T}^1\left(f_t + (1-f_t)\frac{\bar{P}_{t,T}^2}{\bar{P}_{t,T}^1}\right) \\
&= f_t\bar{P}_{t,T}^1 + (1-f_t)\bar{P}_{t,T}^2 = \bar{P}_{t,T}^0.
\end{aligned}$$

The statements about the break-even inflation rate remain to be proven.

Suppose $\gamma = 1$. We know from Theorem 1 that $B_{t,T} = \bar{B}_{t,T}^0$ if $\gamma = 1$. Similarly, we know from Theorem 4 that $P_{t,T} = \bar{P}_{t,T}^0$ if $\gamma = 1$. Hence, $\frac{P_{t,T}}{B_{t,T}} = \frac{\bar{P}_{t,T}^0}{\bar{B}_{t,T}^0}$ and, thus, the break-even inflation rate does not depend on disagreement.

Counterexamples: Figure 8 shows the difference between the break-even inflation rate in an economy with and without inflation disagreement as a function of risk aversion. The price level today is normalized to one. In the high inflation state, it is 1.25. In the low inflation state, it is 0.9. The second investor thinks that both inflation states are equally likely. Suppose the first investor thinks that the probability of a high inflation state is less likely than the second investor thinks. The red area shows that the break-even inflation rate is lower with disagreement if $\gamma > 1$ and higher if $\gamma < 1$. Suppose the first investor thinks that the probability of a high inflation state is more likely than the second investor thinks. The blue area shows that the break-even inflation rate is higher with disagreement if $\gamma > 1$ and lower if $\gamma < 1$.

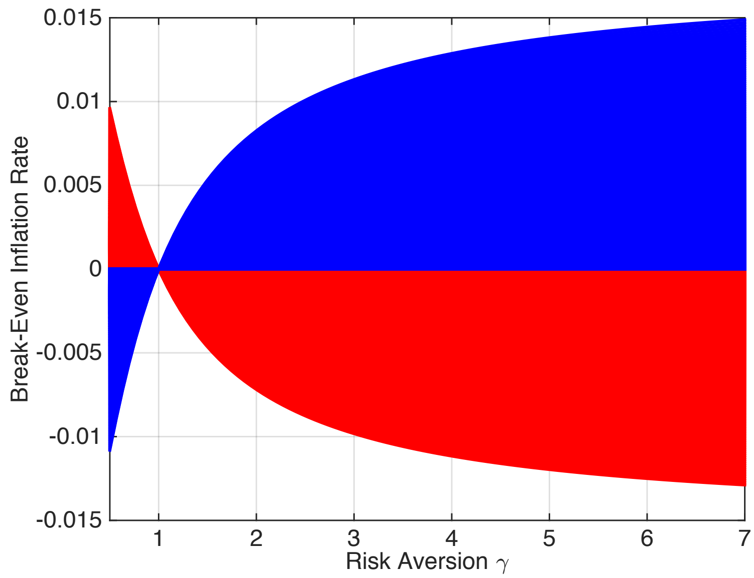

Figure 8: Break-Even Inflation Rate in Edgeworth Box

This plot shows the difference between the break-even inflation rate in an economy with and without inflation disagreement as a function of risk aversion. The price level today is normalized to one and it is 1.25 in the high and 0.9 in the low inflation state tomorrow. The second investor thinks that both inflation states are equally likely.

□

Proof of Theorem 3. Assumption 1 implies Assumption 3. Moreover, equation (A.5) is satisfied if Assumption 2 holds. Hence, Theorem 3 follows from Theorem 4. □

B The SPF-Based GDP Deflator

We briefly explain the GDP deflator data from the Survey of Professional Forecasters (SPF) that we use to calculate disagreement about the mean, variance, and skewness of one-year inflation. Specifically, the survey asks professional forecasters each quarter to assign probabilities to a set of fixed bins for GDP deflator growth until the end of this year and the end of next year. Hence, the forecast horizon is decreasing throughout the current and following year.[50] To determine a probability distribution for one year inflation rates, we interpolate between both forecasts. Specifically, for forecaster j we approximate the fixed horizon forecast in the following way:

$$x^j = w_{quarter} x^j_{current} + (1 - w_{quarter}) x^j_{next}, \tag{B.1}$$

where $x^j_{current}$ is the forecast for the current year, x^j_{next} is the forecast for the next year, and $w_{quarter} \in \{1, 2/3, 1/3, 0\}$ are the weights. For each forecaster, we construct the implied mean, variance, and skewness based on the histograms. Specifically, we assume that for a specific bin all the probability mass is concentrated at the mid-point. Let there be N bins with x_n the mid-point of bin n. For forecaster $j = 1, \ldots, J$ the mean, variance, and skewness are

$$m_j = \sum_{n=1}^{N} p^j_n x_j, \tag{B.2}$$

$$v_j = \sum_{n=1}^{N} p^j_n x_j^2 - m_j^2, \tag{B.3}$$

$$sk_j = \frac{\sum_{n=1}^{N} p^j_n x_j^3 - 3 m_j v_j - m_j^3}{v_j^{\frac{2}{3}}}, \tag{B.4}$$

where p^j_n is the probability mass assigned to bin n by forecaster j and m_j, v_j, and sk_j are the mean, variance, and skewness of the inflation distribution for forecaster j, respectively. Given a cross section of J forecasters at time t, we calculate disagreement about the mean, variance, and skewness of inflation as the cross-sectional standard deviation of the individual mean, variance, and skewness forecasts.

C Model-Based Quantitative Evidence

Proof of Proposition 2. The disagreement measure is

$$\mathcal{D}_{t,T} = \frac{1}{2(T-t)} \mathbb{E}^1 \left[\int_t^T \Delta_s^2 ds \right] = \frac{1}{2(T-t)} \int_t^T \mathbb{E}^1 \left[\Delta_s^2 \right] ds$$

To evaluate the above we need to know $E^1 [\Delta_s^2]$. To this end, note that by Ito's lemma

$$d\Delta_t^2 = 2\beta \left(\frac{\sigma_\Delta^2}{2\beta} - \Delta_t^2 \right) dt - 2\beta \Delta_t dz^1_{\Pi,t}.$$

[50]Our time series starts in Q3 of 1981 because prior to that forecasters only assigned probabilities to GDP growth in the current year.

Using the dynamics of Δ_t^2, we have $\mathbb{E}^1\left[\Delta_s^2\right] = \frac{\sigma_\Delta^2}{2\beta} + e^{-2\beta}\left(\Delta_t^2 - \frac{\sigma_\Delta^2}{2\beta}\right)$. Inserting this back into the expression for the disagreement measure and integrating yields the result. □

Proof of Proposition 3. Assume γ is integer. The real bond price is $B_{t,T} = \mathbb{E}_t^1\left[\frac{\xi_T^1}{\xi_t^1}\right]$. From Proposition 1, we have that the SDF is

$$\begin{aligned}\xi_t^1 &= (y^1)^{-1}e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}f(\lambda_t)^{-\gamma} = (y^1)^{-1}e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}\left(1+(y\lambda_t)^{\frac{1}{\gamma}}\right)^{\gamma} \\ &= \sum_{k=0}^{\gamma}\binom{\gamma}{k}(y^1)^{-1}e^{-\rho t}C_t^{-\gamma}H_t^{\gamma-1}(y\lambda_t)^{\frac{k}{\gamma}}.\end{aligned}$$

Inserting the above into the expression for the bond price we have

$$\sum_{k=0}^{\gamma} w_t^k \mathbb{E}_t^1\left[\left(\frac{C_T}{C_t}\right)^{-\gamma}\left(\frac{H_T}{H_t}\right)^{\gamma-1}\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{k}{\gamma}}\right], \quad \text{where} \quad w_t^k = \binom{\gamma}{k}\frac{\lambda_t^{\frac{k}{\gamma}}}{\left(1+\lambda_t^{\frac{1}{\gamma}}\right)^{\gamma}}.$$

Define $\frac{\xi_T^k}{\xi_t^k} = \left(\frac{C_T}{C_t}\right)^{-\gamma}\left(\frac{H_T}{H_t}\right)^{\gamma-1}\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{k}{\gamma}}$. We can think of this as a stochastic discount factor in an artificial economy. Applying Ito's lemma we have

$$\frac{d\xi_t^k}{\xi_t^k} = -r_t^k dt - \theta_t^k dz, \quad \text{where} \quad dz = \left(dz_{C,t}, dz_{\Pi,t}^1\right)$$

and

$$\theta_t^k = \left(\gamma\sigma_C, \frac{k}{\gamma}\Delta_t\right), \quad \text{and} \quad r_t^k = \rho + \gamma\mu_C - \frac{1}{2}\gamma(\gamma+1)\sigma_C^2 - \delta(\gamma-1)\omega_t + \frac{1}{2}\frac{k}{\gamma}\left(1-\frac{k}{\gamma}\right)\Delta_t^2.$$

Define the state vector $Y_{1,t} = (\Delta_t, \omega)$. We have that $Y_{1,t}$ follows a multidimensional Ornstein-Uhlenbeck process. Moreover, the real short rate in the artificial economies are quadratic in the state vector and the market prices of risk are linear in the state vector. Hence, the artificial state price densities are in the class of quadratic Gaussian term structure (QGTS) models and the solution to $\mathbb{E}_t^1\left[\left(\frac{C_T}{C_t}\right)^{-\gamma}\left(\frac{H_T}{H_t}\right)^{\gamma-1}\left(\frac{\lambda_T}{\lambda_t}\right)^{\frac{k}{\gamma}}\right] = \mathbb{E}^1\left[\frac{\xi_T^k}{\xi_t^k}\right]$ is an exponential quadratic function of the state vector with time dependent coefficients that are solutions to ordinary differential equations.[51]

□

Proof of Proposition 4. The proof follows similar steps as in the proof of Proposition 3. In

[51]We derive solutions to bond prices that belong to the class of QGTS models in the Internet Appendix.

particular, the bond price can be written as

$$\sum_{k=0}^{\gamma} w_t^k \mathbb{E}_t^1 \left[\left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \frac{\Pi_t}{\Pi_T} \right],$$

and we can define a set of artificial nominal stochastic discount factors

$$\frac{\xi_{\Pi,T}^k}{\xi_{\Pi,t}^k} = \left(\frac{C_T}{C_t} \right)^{-\gamma} \left(\frac{H_T}{H_t} \right)^{\gamma-1} \left(\frac{\lambda_T}{\lambda_t} \right)^{\frac{k}{\gamma}} \frac{\Pi_t}{\Pi_T}.$$

Applying Ito's lemma, we have

$$\frac{d\xi_{\Pi,t}^k}{\xi_{\Pi,t}^k} = -r_{\Pi,t}^k dt - \theta_{\Pi,t}^k dz, \quad \text{where} \quad \theta_{\Pi,t}^k = \theta_t^k + \sigma_\Pi, \quad r_{\Pi,t}^k = r_{\Pi,t}^k + x_t^1 + \frac{k}{\gamma}\Delta_t - \sigma_\Pi^2.$$

Define the state vector $Y_t = (x_t^1, \Delta_t, \omega)$. We have that Y_t follows a multidimensional Ornstein-Uhlenbeck process. Moreover, the real short rate in the artificial economies are quadratic in the state vector and the market prices of risk are linear in the state vector. Hence, the artificial state price densities are in the class of QGTS models and, thus, we can solve for the bond price in closed form up to the solution of ordinary differential equations. □