Commodity Financialization: Risk Sharing and Price Discovery in Commodity Futures Markets

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Abstract

We study how commodity financialization affects trading behavior, prices, and welfare through affecting risk sharing and price discovery in futures markets. Our analysis highlights a supply channel through which the futures price feeds back into the later spot price. This feedback effect tends to reduce price efficiency but improve welfare. Consistent with recent evidence, we show that financial traders either provide or demand liquidity in the futures market, depending on the information environment, and that commodity financialization reduces the futures price bias through broadening risk sharing and injecting information into the market. Commodity financialization also has important welfare implications.

Keywords: Commodity financialization, supply channel, liquidity providers/demanders, futures price bias, welfare, price efficiency

JEL Classifications: D82, G14

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1 Introduction

Historically, futures market was introduced for commodity producers (such as farmers) and demanders (such as manufacturers) to share later spot-price risks and control costs. Over the past decade, particularly after the year of 2004, commodity futures have become a popular asset to financial investors, such as hedge funds and commodity index traders. This process has been referred to as the "financialization of commodity markets" (Basak and Pavlova, 2014; Cheng and Xiong, 2014). The fundamental role of futures market is to facilitate risk sharing and price discovery. Economists and regulators are concerned about whether and how financialization has affected the functioning of commodity futures markets.

According to the 2011 Report of the G20 Study Group on Commodities (p. 29), "(t)he discussion centers around two related questions. First, does increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility? And second, does financial investment in commodity futures affect spot prices?" In addition, the existing empirical literature has asked many other important questions. For instance, do financial traders provide liquidity or demand liquidity in the futures market? How does the presence of more financial traders affect the futures price biases (i.e., the deviations of the future price from the expected later spot price)? In this paper, we develop a rational expectations equilibrium model to answer these questions.

Our model features one commodity good (labeled as "wheat") and two periods ($t = 0$ and 1). At date 1, there is a wheat spot market where consumers as commodity demanders trade against commodity suppliers (labeled as "growers," who represent commercial hedgers in reality). The resulting spot price is determined by two shocks, a demand shock $\tilde{\theta}$ built in consumers' preference and a supply shock $\tilde{c}$ built in growers' production technology. At date 0, the growers and K financial traders trade futures contracts, where financialization is parameterized by the number K of financial traders active in the futures market. The futures market features asymmetric information—growers as wheat producers are informed of the supply shock $\tilde{c}$, while financial traders have diverse private signals about the demand shock $\tilde{\theta}$. Growers trade futures to hedge their wheat production and to speculate on their private information. Financial traders trade futures for speculation purposes.

Our analysis highlights a supply channel through which the futures price affects the later spot price: a higher futures price induces growers to supply more commodity, which in turn drives down the spot price through the spot market clearing mechanism. In our setting, growers actively learn from the futures price to guide their real production decisions. In effect, because both the futures contract and their production technology expose them to the same source of risks, growers can effectively treat the futures price as their wheat selling price in making production decisions. As a result, through affecting the futures price and hence growers' commodity supply, financial investment in commodity futures can affect spot prices. Thus, our analysis provides a positive answer to G20's second question.

The supply channel also provides a natural setting for the "feedback effect" studied in the finance literature, which refers to the phenomenon that the price of a traded asset affects its cash flow (see Bond, Edmans, and Goldstein (2012) for a survey). In our setting, the traded asset is the futures contract whose cash flow is the later spot price. Thus, through the supply channel, the price of the futures contract naturally feeds back to its own cash flow. In Section 7, we show that this feedback effect tends to reduce market efficiency, which is a central topic in financial economics and concerns how much the asset price reflects its cash flow. Nonetheless, the feedback effect can benefit all agents of the economy, which highlights the delicate link between market efficiency and social welfare.

We then use our framework to examine various issues that have been of interests to the empirical literature. First, we find that financial traders can either provide or demand liquidity in the futures market, depending on the information environment. In addition, financial traders may change faces in the process of financialization, for instance, they may switch from liquidity providers to demanders as there are more of them in the futures market. These results help to reconcile the mixed empirical evidence that financial traders both provide and demand liquidity in commodity futures market. For instance, Moskowitz, Ooi, and Pedersen (2012) argue that their finding is consistent with that financial traders as speculators provide liquidity. By contrast, according to Kang, Rouwenhorst, and Tang (2014), financial traders demand liquidity from hedgers in commodity futures market. Cheng, Kirilenko, and Xiong (2014) document that financial traders can either provide or demand liquidity at different time periods. This evidence is broadly consitent with our theoretical analysis.

Second, we show that the futures market can either feature a normal backwardation (i.e., a downward bias in futures price relative to the expected value of the later spot price) or a contango (i.e., an upward bias in future price). When the average production cost is relatively low, a normal backwardation ensues, and otherwise, a contango follows. We then demonstrate that commodity financialization reduces the futures price bias through two effects. The direct effect is that more financial traders in the futures market can share the risk off-loaded by growers. The indirect effect is that more financial traders bring more new information to the market, which, through the futures price, reduces the risks faced by all market participants. Consistent with our finding, Hamilton and Wu (2014) have recently documented that the risk premium in crude oil futures on average decreased since 2005.

Finally, our framework delivers normative implications and thus informs regulators of who wins and who loses in the process of commodity financialization. Specifically, as more financial traders come to the futures market, the welfare of commodity consumers increases, while the welfare of each individual financial trader decreases. The welfare implication for growers is ambiguous. On the one hand, more financial traders can share risks off-loaded by the hedging activities of growers, thereby benefiting growers. On the other hand, commodity financialization reduces the futures price bias, which lowers the trading gains that come from growers's speculation activities based on private information. The trade-off between these two offsetting effects implies a non-monotone relation between growers' welfare and the number of financial traders in the futures market.

2 Related Literature

2.1 Literature on commodity financialization

So far, the literature on financialization of commodities is largely empirical and it documents the trading behavior of financial traders in futures markets and their pricing impact.[1] The theoretical research is relatively scarce. Basak and Pavlova (2014) and Baker (2014)

[1] See Irwin and Sanders (2011) and Cheng and Xiong (2014) for excellent surveys.

construct dynamic equilibrium models to study asset price dynamics. While their analyses offer important insights, their models feature symmetric information, which is therefore not suitable for our goal of analyzing how financialization affects price discovery in futures markets. In addition, we consider some dimensions that their models do not (such as welfare implications) and their models consider some dimensions that we do not (such as return dynamics), and therefore, our analysis complements theirs.

In a contemporaneous paper, Leclercq and Praz (2014) have also explored how the entry of new speculators affects the supply of the commodity in a rational expectations equilibrium model. Our model differs from and complements theirs in two important ways. First, in Leclercq and Praz's (2014) setup, the equilibrium futures price is equal to the exogenous commodity production cost (due to an assumption of a linear cost technology) and commodity producers do not learn from the futures price, which largely limits the information aggregation role of the futures price. In contrast, in our setup, the futures price aggregates both demand and supply side information and commodity producers actively learn from asset prices to guide their real production decisions. This difference makes our model able to characterize the role of the feedback effect in Section 7, which is not feasible in Leclercq and Praz (2014). Second, our positive analysis has explored additional important topics studied in the existing empirical literature—for instance, whether financial traders provide or demand liquidity, how commodity financialization affects futures price bias—while Leclercq and Praz's (2014) focus is on the average and volatility of spot prices.

In addition, Goldstein, Li, and Yang (2014) and Sockin and Xiong (2015) develop asymmetric information models to explore the implications of the financialization of commodity futures markets. Our paper and those studies complement each other in many important dimensions. Goldstein, Li, and Yang (2014) emphasize that in the futures market, because financial institutions are limited to trade in the futures contracts for speculation purposes, while commodities producers trade the futures contracts mostly for hedging, these two groups of traders they may respond to the *same* information in opposite directions. This can lead to a reduction in price informativeness and an increase in the futures risk premium. By contrast, in our setting, financial institutions and commodities producers observe *different* information, and commodity financialization always reduces the futures price biases.

Sockin and Xiong (2015) focus on information asymmetry in the spot market, and their main theory insight is that a high spot price may further spur the commodity demand through an informational channel, which may therefore fuel a short-term bubble in spot prices. In contrast, our analysis focuses on information asymmetry in the futures market and the real effect on spot prices is through a supply channel in our setting. Moreover, we have provided both positive and normative analyses, while both Goldstein, Li, and Yang (2014) and Sockin and Xiong (2015) mainly focus on pricing implications.

2.2 Literature on futures markets

The literature on futures market is classic and huge (see Section 1.1 of Acharya, Lochstoer and Ramadorai (2013) for a brief literature review on this literature). This literature has developed theories of "hedging pressure" (Keynes, 1930; Hirshleifer, 1988, 1990) or "storage" (Kaldor, 1939; Working, 1949) to explain futures prices. Notably, the literature has also developed asymmetric information models on futures market (e.g., Grossman, 1977; Danthine, 1978; Bray, 1981; Stein, 1987). However, because commodity financialization is just a recent phenomenon, these models have focused on different research questions, for instance, on whether the futures market is viable (Grossman, 1977), on whether the futures price is fully revealing (Danthine, 1978; Bray, 1981), and on whether speculative trading can reduce welfare (Stein, 1987).

Among these models, Stein (1987) is closest in terms of research topics. Stein (1987) shows that introducing a new asset can harm welfare by generating price volatility. However, he does not explore the general implications of financialization for risk sharing and price discovery, such as the effect on the trading behavior of financial traders. Also, his model features an endowment economy, while our setup has commodity production, which is crucial for many of our results.

In terms of model setup, our model is closer to Danthine (1978). His main finding is that the futures price provides a sufficient statistics used by rational traders in formulating their probability distributions. The futures price is not fully revealing in our setup, because we have introduced two sources of uncertainty, similar to Bray (1981) and Goldstein and Yang (2015). In addition, our analysis has focused on topics emphasized by the recent empirical literature on financialization of commodities.

3 The Model

We consider an asymmetric information model similar to Grossman (1977), Danthine (1978), and Stein (1987) to study the implications of commodity financialization. There are two consumption goods—a commodity good (call it wheat) and a numeraire good (call it money)—and two tradable assets, a futures contract on the commodity and a risk-free asset (with the net risk-free rate normalized at zero). Time lasts two periods: $t = 0$ and 1. At date 0, $K \in \mathbb{N}$ financial investors, such as hedge funds or commodity index traders, trade futures contracts against $J \in \mathbb{N}$ commodity producers (call them growers). Here we use parameter K to capture financialization of commodities, i.e., the process of commodity financialization corresponds to an increase in K. Growers make their investment on the commodity production at date 0, which in turn determines the commodity supply at the date-1 spot market. At date 1, $J \in \mathbb{N}$ consumers purchase commodity from the spot market, and all agents consume and exit the economy.[2] The timeline of the economy is summarized in Figure 1. We next describe in detail the behavior and information structure of each type of agents.

3.1 Consumers

There are $J > 0$ identical consumers. At date 1, a representative consumer derives utility from the two consumption goods according to the following utility function:

$$U_C(y, m) = -\frac{1}{2}y^2 + \tilde{\theta}y + m, \tag{1}$$

where y is the wheat consumption, m is the left-over money, and $\tilde{\theta}$ denotes a preference shock which follows a normal distribution with a mean of $\bar{\theta} \in \mathbb{R}$ and a precision (the inverse of variance) of $\tau_\theta > 0$ (i.e., $\tilde{\theta} \sim N\left(\bar{\theta}, 1/\tau_\theta\right)$).[3] When making consumption choices, consumers know the preference shock $\tilde{\theta}$ and the spot price $\tilde{v}$, which differs from Sockin and Xiong (2015) whose analysis emphasizes the information inference problem in commodity

[2] For simplicity, we have assumed that the number of consumers and the number of growers are the same. This assumption is just a normalization without loss of generality.

[3] Throughout the paper, we use a tilde ($\tilde{}$) to signify a random variable, where a bar denotes its mean and τ denotes its precision. That is, for a random variable $\tilde{z}$, we have $\bar{z} \equiv E(\tilde{z})$ and $\tau_z = \frac{1}{Var(\tilde{z})}$.

demanders' decisions.

Preference shock $\tilde{\theta}$ can also be interpreted as a technology shock. For instance, suppose that consumers do not directly consume wheat and they have to transform wheat into bread according to a concave technology, $B = -\frac{1}{2}y^2 + \tilde{\theta}y$, where B is the bread output and y is the wheat input. Then preference (1) follows directly. We are agonistic about both interpretations. The key is that $\tilde{\theta}$ represents demand shocks in the commodity spot market. We also follow Stein (1987) and assume that consumers do not trade in the futures market back at date 0.

The behavior of consumers generates the commodity demand in the spot market. Their effective role in the model is to provide a parsimonious device that determines the wheat spot price. Specifically, let $\tilde{v}$ denote the spot price of wheat and normalize each consumer's initial endowment at 0. Then, a representative consumer's problem is

$$\max_{y} \left(-\frac{1}{2}y^2 + \tilde{\theta}y - \tilde{v}y \right),$$

which yields the following wheat demand function of each consumer:

$$y = \tilde{\theta} - \tilde{v}. \tag{2}$$

The aggregate wheat demand is $J \times y = J\left(\tilde{\theta} - \tilde{v}\right)$. The spot price $\tilde{v}$ is random and its distribution will be endogenously derived in subsequent sections.

The essential role of consumers in our model is to provide the wheat demand function (2) to determine the spot price $\tilde{v}$.[4] In this sense, consumers may be better interpreted as those parties in reality who deal with physical commodities but do not participate in the futures market (e.g., some airline companies) because of market frictions such as limited knowledge about the operation of futures markets.

3.2 Growers

There are $J > 0$ identical growers who only consume the numeraire good. We make growers risk averse to introduce their hedging motives. Specifically, a representative grower has a constant absolute risk aversion (CARA) utility with a risk aversion coefficient of $\alpha > 0$; that

[4] Specifically, growers' information set does not include the later spot price $\tilde{v}$ and thus their decisions on wheat production x does not depend on $\tilde{v}$ (see equation (9)). It is consumers' wheat demand that brings $\tilde{v}$ into the market clearing condition of the spot market.

is, his utility function is $U_G(W) = -e^{-\alpha W}$, where W is his final wealth at the end of date 1. Growers make decisions at date 0 and these decisions are twofold. First, they decide how much wheat to produce, which will in turn determine the wheat supply at the date-1 spot market. Second, they decide how many futures contracts to invest in the futures market to hedge their wheat production and to speculate on their private information that will be introduced shortly.

Consider a representative grower. If he decides to produce x units of wheat, he needs to pay a production cost[5]

$$C(x) = \tilde{c}x + \frac{1}{2}x^2, \tag{3}$$

where the cost shock $\tilde{c} \sim N(\bar{c}, 1/\tau_c)$ (with $\tau_c > 0$) is independent of the demand shock $\tilde{\theta}$. The cost function (3) is convex in x. The cost shock $\tilde{c}$ is growers' private information. We introduce this shock for two reasons. First, it is empirically relevant that commodity producers indeed have valuable private information when they trade futures. Second, we will assume that financial traders possess private information on $\tilde{\theta}$, and by doing so, we will have a structure that financial traders and growers possess different information, so that the interactions between these two dimensional information can endogenously make financial traders either provide or demand liquidity in equilibrium.

The representative grower trades futures contracts. The payoff on the futures contract is the wheat spot price $\tilde{v}$ at date 1. Each unit of futures contract is traded at an endogenous price $\tilde{p}$. This price is observable to all market participants and it contains valuable information. Thus, the grower's information set is $\mathcal{I}_G \equiv \{\tilde{c}, \tilde{p}\}$. His decision is to choose wheat production x and futures investment d_G (and investment in the risk-free asset) to maximize

$$E\left(-e^{-\alpha \tilde{W}_G} \middle| \tilde{c}, \tilde{p}\right) \tag{4}$$

subject to

$$\tilde{W}_G = \tilde{v}x - \tilde{c}x - \frac{1}{2}x^2 + (\tilde{v} - \tilde{p})\, d_G, \tag{5}$$

where we have normalized grower's initial endowment as 0.

[5] The cost function $C(x)$ can be alternatively interpreted as an inventory cost. For instance, suppose that the date-0 wheat spot price is v_0 and carrying an inventory of x units of wheat incurs a cost of $\tilde{c}x + \frac{1}{2}x^2$. Then the total cost of storing x units of wheat is $C(x) = (\tilde{c} + v_0)\, x + \frac{1}{2}x^2$, which is essentially equation (3) with a renormalization of the mean of $\tilde{c}$. However, this interpretation is made in a partial equilibrium setting as the date-0 spot price v_0 is exogenous. We can fully endogenize this spot price as well at the expense of introducing one extra source of uncertainty, because otherwise the prices of futures and current spot price will combine to fully reveal the shocks (see Grossman, 1977).

To better connect our setup to previous models, we have followed the literature (e.g., Danthine, 1978) and interpreted growers as commodity suppliers. In effect, a more precise interpretation of growers should be commercial hedgers, because as become clear later, their futures demand contains a hedging component (see equation (12)). In this sense, growers can be either commodity providers or commodity demanders. Specifically, if $x < 0$, then in equation (5) the term $\tilde{v}x - \tilde{c}x - \frac{1}{2}x^2$ can be interpreted as the utility from consuming $|x|$ units of wheat, and so growers are wheat demanders. We can also extend the model to have multiple growers who receive different cost shocks, so that in equilibrium some growers supply wheat while others demand wheat. We abstract from this extension for simplicity.

3.3 Financial traders

There are $K \geq 0$ financial traders who derive utility only from their consumption of the numeraire good at date 1. They have a CARA utility with a risk aversion coefficient of $\gamma > 0$. Financial traders trade futures only for speculation, not for hedging any real production of commodities. To capture the fact that financial traders can bring new information to the market, we assume that they have private information about the demand shock $\tilde{\theta}$ in the later spot market (recall that growers have private information about the supply shock $\tilde{c}$). In addition, we specify that financial traders have diverse information to implement the notion that as more financial traders come to the market, they bring more information in aggregate. That is, at date 0, prior to trading, a financial trader k receives a private signal

$$\tilde{s}_k = \tilde{\theta} + \tilde{\varepsilon}_k \text{ with } \tilde{\varepsilon}_k \sim N\left(0, 1/\tau_\varepsilon\right) \text{ and } \tau_\varepsilon \geq 0, \tag{6}$$

where $\tilde{\varepsilon}_k$ is independent of each other and of other random variables. Parameter τ_ε controls the quality of the private signals. We allow the possibility of $\tau_\varepsilon = 0$ which corresponds to the case in which financial traders do not have any private information at all.

Financial traders also observe the futures price $\tilde{p}$, and thus financial trader k has an information set $\mathcal{I}_{F,k} \equiv \{\tilde{s}_k, \tilde{p}\}$. His decision problem is to choose investments in futures and the risk-free asset to maximize the conditional expected utility. Specifically, let $d_{F,k}$ be the futures demand of financial trader k, and his decision problem is

$$\max_{d_{F,k}} E\left[-e^{-\gamma(\tilde{v}-\tilde{p})d_{F,k}} \middle| \tilde{s}_k, \tilde{p}\right], \tag{7}$$

where we have also normalized the initial endowment of financial traders to be zero. For simplicity, we have assumed that financial traders do not trade other assets such as stocks. As long as the payoffs of other assets are independent of futures payoff, our results will not change even if more assets are introduced into the economy. However, if the payoffs are correlated across assets, financial traders may trade futures also for hedging motives, which will complicate the analysis significantly. We do not expect this modification changes the main results of the paper.

4 Equilibrium Characterization

An equilibrium consists of two subequilibria: the date-1 spot market equilibrium and the date-0 futures market equilibrium. At date 1, consumers maximize their preferences, which yields the wheat demand function and in turn clears the wheat supply provided by growers at the prevailing spot price $\tilde{v}$. Because the wheat demand depends on the demand shock $\tilde{\theta}$ and the wheat supply depends on the cost shock $\tilde{c}$ and futures price $\tilde{p}$, the spot price $\tilde{v}$ will be a function of $\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$. At date 0, we consider a competitive rational expectations equilibrium (REE) in the futures market. Given that growers have private information $\tilde{c}$ and financial traders have private information $\{\tilde{s}_k\}_{k=1}^{K}$, the futures price $\tilde{p}$ will depend on $(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K)$, leading to the futures price function $p(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K)$. Growers and financial traders extract information from observing $\tilde{p}$ as well as their own private information to maximize their expected utilities at the prevailing price.

Definition 1 *An equilibrium consists of a spot price function, $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right) : \mathbb{R}^3 \to \mathbb{R}$, a futures price function, $p(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K) : \mathbb{R}^{K+1} \to \mathbb{R}$, a commodity production policy, $x(\tilde{c}, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, a trading strategy of growers, $d_G(\tilde{c}, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, and a trading strategy of financial traders, $d_F(\tilde{s}_k, \tilde{p}) : \mathbb{R}^2 \to \mathbb{R}$, such that:*
(a) at date 1, the spot market clears, i.e., $J \times \left[\tilde{\theta} - v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)\right] = J \times x(\tilde{c}, \tilde{p}) \Longleftrightarrow \tilde{\theta} - v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right) = x(\tilde{c}, \tilde{p})$;
(b) at date 0, given that $\tilde{v}$ is defined by $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$, (b1) $x(\tilde{c}, \tilde{p})$ and $d_G(\tilde{c}, \tilde{p})$ solve for growers' problem given by (4) and (5); (b2) $d_F(\tilde{s}_k, \tilde{p})$ solves financial traders' problem (7); and (b3) the futures market clears, i.e., $J \times d_G(\tilde{c}, \tilde{p}) + \sum_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}) = 0$.

We next construct an equilibrium in which the price functions $v\left(\tilde{\theta}, \tilde{c}, \tilde{p}\right)$ and $p\left(\tilde{c}, \tilde{s}_1, ..., \tilde{s}_K\right)$ are linear. As standard in the literature, we solve the equilibrium backward from date 1.

4.1 Date-1 spot market equilibrium

The wheat demand is given by equation (2), $y = \tilde{\theta} - \tilde{v}$. The wheat supply is determined by growers' date-0 investment decisions. Solving growers' problem (given by (4) and (5)) yields the following first-order conditions:

$$\begin{aligned} x + d_G &= \frac{E\left(\tilde{v}|\mathcal{I}_G\right) - \tilde{p}}{\alpha Var\left(\tilde{v}|\mathcal{I}_G\right)}, &(8)\\ x &= \tilde{p} - \tilde{c}. &(9)\end{aligned}$$

The above expressions are similar to those in Danthine (1978). The intuition is as follows. Given both real investment x and financial investment d_G expose a grower to the same risk source $\tilde{v}$, his overall exposure to this risk is given by the standard demand function of a CARA-investor, as expressed in (8). Expression (9) says that after controlling the total risk given by (8), growers essentially treat $\tilde{p}$ as the wheat selling price when making real production decisions. Aggregating (9) across all growers delivers the aggregate wheat supply at the spot market: $J \times x = J\left(\tilde{p} - \tilde{c}\right)$. By the market clearing condition $y = x$ and equations (2) and (9), we can solve the spot price $\tilde{v}$, which is given by the following lemma.

Lemma 1 *The date-1 spot price $\tilde{v}$ is given by*

$$\tilde{v} = \tilde{\theta} + \tilde{c} - \tilde{p}. \tag{10}$$

This lemma formally establishes a supply channel through which the date-0 futures price $\tilde{p}$ affects the date-1 spot price $\tilde{v}$. Equation (10) therefore provides a positive answer to the following question raised in the 2011 G20 Report on Commodities, "does financial investment in commodity futures affect spot prices?" That is, financial traders' futures investments will alter the behavior of $\tilde{p}$, which in turn changes the later spot price $\tilde{v}$ through equation (10). In other words, the futures market is not just a side show, and it has consequences for the real side. This phenomenon is labeled as the "feedback effect" in the finance literature, i.e., the price $\tilde{p}$ of a traded asset feeds back to its own cash flows $\tilde{v}$ (recall that for a futures contract, its cash flow is the later spot price).[6]

[6]See Bond, Edmans, and Goldstein (2012) for a survey on the feedback effect literature.

Lemma 1 has two additional interesting implications. First, the feedback effect identified in our setting tends to weaken the correlation between $\tilde{p}$ and $\tilde{v}$, $Cov(\tilde{p}, \tilde{v})$. This correlation is usually viewed as a measure for informational efficiency of the financial market, i.e., how well the market price $\tilde{p}$ reflects its cash flows $\tilde{v}$. We will explore this implication later in Section 7. Second, if we hold constant fundamental shocks $\tilde{\theta}$ and $\tilde{c}$, the noise terms $(\tilde{\varepsilon}_1, ..., \tilde{\varepsilon}_2)$ in financial traders' private signals can cause the futures price $\tilde{p}$ and the later spot price $\tilde{v}$ to move in opposite directions. That is, $Cov\left(\tilde{p}, \tilde{v}|\tilde{\theta}, \tilde{c}\right) < 0$. This means that under those realization paths without fundamental changes in the supply and demand sides, we may observe a negative correlation between futures and spot prices driven by financial activities.

4.2 Date-0 futures market equilibrium

We conjecture the following futures price function:

$$\tilde{p} = p_0 + p_s \tilde{S} + p_c \tilde{c} \text{ with } \tilde{S} \equiv \frac{1}{K} \sum\nolimits_{k=1}^{K} \tilde{s}_k, \tag{11}$$

where p_0, p_c, and p_s are undetermined coefficients. Next, we compute the demand function of futures market participants and use the market clearing condition to construct such a linear REE price function.

By (8) and (9), growers' demand for the futures contract is

$$d_G(\tilde{c}, \tilde{p}) = \underbrace{\frac{E(\tilde{v}|\mathcal{I}_G) - \tilde{p}}{\alpha Var(\tilde{v}|\mathcal{I}_G)}}_{\text{speculation}} - \underbrace{(\tilde{p} - \tilde{c})}_{\text{hedging}}. \tag{12}$$

Thus, growers trade futures for two reasons. First, they hedge their real commodity production of $x = \tilde{p} - \tilde{c}$. Second, because they also have private information on their production cost $\tilde{c}$, they speculate on this private information. Given that growers' information set is $\mathcal{I}_G \equiv \{\tilde{c}, \tilde{p}\}$, they can use the price function (11) to back out signal $\tilde{S}$ (provided $p_s \neq 0$), which provides information about the demand component $\tilde{\theta}$ in the later spot price $\tilde{v}$. Formally, growers' conditional forecast is

$$E(\tilde{v}|\mathcal{I}_G) = E\left(\tilde{\theta}\middle|\tilde{S}\right) + \tilde{c} - \tilde{p} = \frac{\tau_\theta \bar{\theta} + K\tau_\varepsilon \tilde{S}}{\tau_\theta + K\tau_\varepsilon} + \tilde{c} - \tilde{p}, \tag{13}$$

$$Var(\tilde{v}|\mathcal{I}_G) = Var\left(\tilde{\theta}\middle|\tilde{S}\right) = \frac{1}{\tau_\theta + K\tau_\varepsilon}. \tag{14}$$

Solving financial trader k's problem in (7), we can compute his futures demand as follows:

$$d_F(\tilde{s}_k, \tilde{p}) = \frac{E(\tilde{v}|\mathcal{I}_{F,k}) - \tilde{p}}{\gamma Var(\tilde{v}|\mathcal{I}_{F,k})}. \tag{15}$$

Clearly, financial trader k trades only for speculating on information $\mathcal{I}_{F,k} \equiv \{\tilde{s}_k, \tilde{p}\}$. Provided $p_c \neq 0$ (which is true in equilibrium), the price $\tilde{p}$ is equivalent to the following signal in predicting $\tilde{v}$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0}{p_c} = \rho \tilde{S} + \tilde{c} \text{ with } \rho \equiv \frac{p_s}{p_c}. \tag{16}$$

Using Bayes' law, we can compute his forecast as follows:

$$Var\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right) = \frac{(K-1)\left(\tau_c + \tau_\theta + K\tau_\varepsilon\right)\rho^2 - 2K\tau_\varepsilon\left(K-1\right)\rho + K^2\tau_\varepsilon}{\tau_c\left(K-1\right)\left(\tau_\theta + K\tau_\varepsilon\right)\rho^2 + K^2\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)}, \tag{17}$$

$$E\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right) = \bar{\theta} + \bar{c} + \beta_{F,p}\left(\tilde{s}_p - \rho\bar{\theta} - \bar{c}\right) + \beta_{F,s}\left(\tilde{s}_k - \bar{\theta}\right) - \tilde{p}, \tag{18}$$

where

$$\beta_{F,p} = \frac{K\tau_\varepsilon\left(\tau_c\left(K-1\right)\rho + K\left(\tau_\theta + \tau_\varepsilon\right)\right)}{\tau_c\left(K-1\right)\left(\tau_\theta + K\tau_\varepsilon\right)\rho^2 + K^2\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)}, \tag{19}$$

$$\text{and } \beta_{F,s} = \frac{K\tau_\varepsilon\left(-\left(\tau_\theta + K\tau_\varepsilon\right)\rho + K\tau_\varepsilon\right)}{\tau_c\left(K-1\right)\left(\tau_\theta + K\tau_\varepsilon\right)\rho^2 + K^2\tau_\varepsilon\left(\tau_\theta + \tau_\varepsilon\right)}. \tag{20}$$

Using the market clearing condition,

$$J \times d_G\left(\tilde{c}, \tilde{p}\right) + \sum\nolimits_{k=1}^{K} d_F\left(\tilde{s}_k, \tilde{p}\right) = 0, \tag{21}$$

and the expressions of the demand functions, and comparing coefficients, we can establish the following proposition regarding the existence and uniqueness of REE in the date-0 futures market.

Proposition 1 *There exists a linear REE where the futures price $\tilde{p}$ and the spot price $\tilde{v}$ are given respectively by*

$$\tilde{p} = p_0 + p_s\tilde{S} + p_c\tilde{c},$$

$$\tilde{v} = \tilde{\theta} + \tilde{c} - \tilde{p},$$

where $\tilde{S} \equiv \frac{1}{K}\sum\nolimits_{k=1}^{K} \tilde{s}_k$ and the coefficients p_0, $p_s \geq 0$, and $p_c > 0$ are given in the appendix. The equilibrium is characterized by $\rho \equiv \frac{p_s}{p_c} \in \left[\frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon + \alpha}, \frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon}\right]$, with ρ being determined by

$$\rho = \frac{J\frac{K\tau_\varepsilon}{\alpha} + \frac{K}{\gamma}\frac{K\tau_\varepsilon\left[-\left(\tau_\theta + K\tau_\varepsilon\right)\rho + K\tau_\varepsilon\right]}{(K-1)(\tau_c + \tau_\theta + K\tau_\varepsilon)\rho^2 - 2K\tau_\varepsilon(K-1)\rho + K^2\tau_\varepsilon}}{J\left(\frac{\tau_\theta + K\tau_\varepsilon}{\alpha} + 1\right)}. \tag{22}$$

Moreover, if growers' risk aversion coefficient α is sufficiently small, then the equilibrium is unique within the linear price function class.

5 Trading and Pricing Implications

5.1 Liquidity providers and demanders

The empirical literature on commodity financialization finds that financial traders can either provide liquidity or demand liquidity in the futures market (see Cheng and Xiong, 2014; Cheng, Kirilenko, and Xiong, 2014; Kang, Rouwenhorst, and Tang, 2014). Our model is useful for understanding this phenomenon. In our analysis, we follow Vayanos and Wang (2012) and define liquidity demanders and providers as follows.

Definition 2 *For trader type $t \in \{G, F\}$, if $Cov\left(d_t\left(\mathcal{I}_t\right), \tilde{p}\right) > 0$, then type-t traders are liquidity demanders; that is, they buy the asset when the price goes up and sell the asset when the price goes down. Otherwise, if $Cov\left(d_t\left(\mathcal{I}_t\right), \tilde{p}\right) < 0$, then type-t traders are liquidity providers.*

Intuitively, if a trader demands liquidity, then it is likely that he initiates the trade. Thus, if he wants to buy the asset, he has to offer a price high enough to attract the other side to engage in the trade. Similarly, if he wants to sell the asset, then he has to sell it at a sufficiently low price to convince the other side to buy in the asset. As a result, a liquidity demander's equilibrium order flow tends to be positively correlated with the equilibrium asset price. By the same token, a liquidity suppler's order flow tends to be negatively correlated with the equilibrium price. Given that financial traders trade against growers in our economy, it must be the case that when financial traders provide liquidity, growers demand liquidity, and vice versa.

In our model, the general equilibrium feature endogenously makes financial traders admit either the role of liquidity demanders or of suppliers, depending on the information environment. Loosely speaking, financial traders tend to demand liquidity when the precision τ_ε of their private information is high and/or when the precision τ_c of the cost shock is high, and they tend to supply liquidity when the opposite is true. The intuition is as follows. First, financial traders speculate on their private information $\tilde{s}_k$, and so if their information is very precise (i.e., if τ_ε is very high), they will trade aggressively, and as a result, their order flows will more likely move prices in the same direction. That is, financial traders demand liquidity when τ_ε is high. Second, growers trade futures both for hedging and for speculation, where the strength of their trading motives is captured by the variance $\frac{1}{\tau_c}$ of the cost shock $\tilde{c}$ in their production technology. When $\frac{1}{\tau_c}$ is high (or when τ_c is small), growers' trading incentives are strong, and it is more likely for them to demand liquidity, which implies that in equilibrium financial traders have to provide liquidity on the other side. As a consequence, financial traders tends to provide liquidity when τ_c is small.

Proposition 2 *Financial traders tend to supply liquidity when τ_ε or τ_c are small, and they tend to demand liquidity when τ_c is large. That is, for a given $K < \infty$, we have $Cov\left(d_F\left(\tilde{p}, \tilde{s}_k\right), \tilde{p}\right) < 0$ for small values of τ_ε or τ_c, and $Cov\left(d_F\left(\tilde{p}, \tilde{s}_k\right), \tilde{p}\right) > 0$ for large values of τ_c.*

Figure 2 provides an illustration for Proposition 2 under the parameter configuration $\tau_\theta = \alpha = \gamma = 1$, $J = K = 20$, $\bar{\theta} = 5$, and $\bar{c} = 1$. We use "+" to indicate the region of $(\tau_\varepsilon, \tau_c)$ for which financial traders demand liquidity in equilibrium, while the blank region indicates the values of of $(\tau_\varepsilon, \tau_c)$ for which financial traders supply liquidity. Indeed, we find that financial traders tend to demand liquidity when either τ_ε or τ_c is high and that they tend to provide liquidity when the opposite is true. Thus, our analysis shows that financial traders can either demand or provide liquidity depending on the information environment, even when financial traders always behave as speculators in futures market (that is, their demand function (15) does not have a hedging component). This contrasts with the literature which typically relies on financial traders' hedging need, say due to portfolio concerns, to make them become liquidity demanders (e.g., Cheng, Kirilenko, and Xiong, 2014; Kang, Rouwenhorst, and Tang, 2014).

Figure 3 conducts a different exercise. Here, we fix the values of $(\tau_\varepsilon, \tau_c)$ and examine how the number K of financial traders affects their liquidity provision/demanding behavior. The general patterns depend on the comparison between τ_c and τ_θ. Specifically, in Panel A, we choose $\tau_c = 2$ and $\tau_\theta = 1$, while in Panel B, we choose $\tau_c = 0.5$ and $\tau_\theta = 1$. In both panels, the other parameters are fixed at $\tau_\varepsilon = 0.1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$, and $\bar{c} = 1$. We find that in Panel A, financial traders start to be liquidity providers as K is small, and then they become liquidity demanders when K becomes large. In contrast, in Panel B, financial traders keep providing liquidity independent of the values of K. Note that Panel A of Figure 3 suggests that as more financial traders come to the futures market, it is possible that financial traders may switch from providing liquidity to demanding liquidity.

The result of Figure 3 can be understood as follows. Given that the qualitative difference between Panels A and B happens when K is large, we consider the limiting case of $K \to \infty$. In the limit, the average signal $\tilde{S} \equiv \frac{1}{K}\sum_{k=1}^{K} \tilde{s}_k \to \tilde{\theta}$, and thus growers perfectly know $\tilde{\theta}$ and $\tilde{c}$. As a result, their trade must force the futures price $\tilde{p}$ to fully reveal $\tilde{v}$, that is, $\tilde{p} = \tilde{v} = \frac{\tilde{\theta}+\tilde{c}}{2}$, where the second equality follows from equation (10). Since the price is close to be fully revealing, the speculation component in $d_G(\tilde{c}, \tilde{p})$ is close to $d_F(\tilde{s}_k, \tilde{p})$ except for the different risk aversion coefficients. Thus, the market clearing condition implies

$$\sum_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}) \propto \tilde{p} - \tilde{c} = \frac{\tilde{\theta} + \tilde{c}}{2} - \tilde{c} = \frac{\tilde{\theta} - \tilde{c}}{2}.$$

As a consequence, we have

$$Cov\left(\sum_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}), \tilde{p}\right) \propto Cov\left(\tilde{\theta} - \tilde{c}, \tilde{\theta} + \tilde{c}\right) = \frac{1}{\tau_\theta} - \frac{1}{\tau_c},$$

and thus, $Cov\left(\sum_{k=1}^{K} d_F(\tilde{s}_k, \tilde{p}), \tilde{p}\right) > 0$ if and only if $\tau_c > \tau_\theta$.

5.2 Futures price biases

The literature has looked at "futures price bias," that is, the deviation of the futures price from the expectation of the later spot price, $E(\tilde{v} - \tilde{p})$. A downward bias in the futures price is termed "normal backwardation," while an upward bias in the futures price is termed "contango." A major branch of literature on futures pricing has attributed bias to hedging pressures of commodity producers. Hamilton and Wu (2014) document that the futures price bias in crude oil futures on average decreased since 2005. Our model sheds light on how commodity financialization affects the average futures price, the average spot price, and the resulting futures price bias.

In the appendix, we show that the futures price bias $E(\tilde{v} - \tilde{p})$ is given as follows:

$$E(\tilde{v} - \tilde{p}) = \frac{\bar{\theta} - \bar{c}}{2A + 1}, \tag{23}$$

where

$$A \equiv \frac{1}{\alpha Var\left(\tilde{\theta}\middle|\tilde{S}\right)} + \frac{K}{J}\frac{1}{\gamma Var\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right)}. \tag{24}$$

Parameter A is a normalized capacity of the market to absorb risks. To see this, note that by the demand functions (12) and (15) and the market clearing condition (21), the futures price can be understood as determined by J CARA-investors with risk aversion α and K CARA-investors with risk aversion γ, while the effective supply is $J(\tilde{p}-\tilde{c})$, which is the aggregate hedging pressure from growers. Thus, in (24), the conditional variances $Var\left(\tilde{\theta}\middle|\tilde{S}\right)$ and $Var(\tilde{v}|\tilde{s}_k,\tilde{p})$ are the payoff risks faced by growers and financial traders, respectively. We then adjust the payoff risks by their respective risk aversions to capture the effect of preferences. The second term is also adjusted by a ratio of $\frac{K}{J}$ to capture the fact that there are K financial traders while the total futures supply is proportional to the number J of growers.

By equations (23) and (24), we have $E(\tilde{v}-\tilde{p})>0$ if and only if $\bar{c}<\bar{\theta}$. The intuition is as follows. When the average cost shock $\bar{c}$ is low, growers tend to produce more wheat and thus they will short more futures to hedge their wheat production, thereby depressing the futures price. But this result is non-trivial, because $\tilde{c}$ affects both the futures price $\tilde{p}$ and the later spot price $\tilde{v}$ in the same direction (see Proposition 1). The key observation is that $\tilde{c}$ affects $\tilde{p}$ more than it affects $\tilde{v}$. Fama and French (1987) used 21 commodities to test the futures risk premium hypothesis, and indeed, they found that some markets feature "normal backwardation," while others feature "contango." According to our theory, this difference can be explained by the relative sizes of the average supply shock and the average demand shock.

Increasing the number K of financial traders tends to decrease futures price bias, that is, $\frac{\partial|E(\tilde{v}-\tilde{p})|}{\partial K}<0$, which is consistent with the empirical evidence provided by Hamilton and Wu (2014). This is because the market's aggregate risk bearing capacity A may increase with K through two channels. First, the newly added financial traders directly share the risk that is loaded off from the hedging needs of growers. Second, since financial traders can bring more information into the market, other existing market participants also learn more information about the futures payoff from reading the futures price, which effectively reduces the payoff risks faced by the market.[7]

[7] These two channels nicely echo the remark made by the G20 Study Group in its report on commodities (2011, p. 6): "Greater (financial) investor participation can be expected to enhance the functioning of markets by adding depth and liquidity. This should help producers and consumers to hedge price fluctuation risks. Greater participation of financial investors can also aid the development of long-term commodity futures, which would facilitate risk management and planning over longer time horizons. More generally,

Similarly, we can show

$$E(\tilde{p}) = \bar{c} + \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right) \text{ and } E(\tilde{v}) = \bar{\theta} - \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right), \tag{25}$$

and thus, parameter K also affects the average futures price $E(\tilde{p})$ and the average spot price $E(\tilde{v})$ through its effect on the capacity A of the market to absorb risks. When $\bar{\theta} > \bar{c}$, growers need to hedge a lot of their wheat production so that the effective futures supply is high, the average futures price $E(\tilde{p})$ increases with the risk absorption capacity A. Due to the increased average futures price, the average spot price $E(\tilde{v}) = \bar{\theta} + \bar{c} - E(\tilde{p})$ decreases with A. When $\bar{\theta} < \bar{c}$, the opposite is true.

Proposition 3 *(a) There is a downward bias (i.e., normal backwardation) in futures price relative to the expected value of the later spot price if and only if $\bar{c} < \bar{\theta}$. That is, $E(\tilde{v} - \tilde{p}) > 0$ if and only if $\bar{c} < \bar{\theta}$.*
(b) Suppose the risk aversion α of growers is sufficiently small.
(b1) Commodity financialization decreases futures price bias. That is, $\frac{\partial |E(\tilde{v}-\tilde{p})|}{\partial K} < 0$.
(b2) If $\bar{c} < \bar{\theta}$, then commodity financialization increases average futures price and decreases average spot price. If $\bar{c} > \bar{\theta}$, then the opposite is true. That is, if $\bar{c} < \bar{\theta}$, then $\frac{\partial E(\tilde{p})}{\partial K} > 0$ and $\frac{\partial E(\tilde{v})}{\partial K} < 0$, and if $\bar{c} > \bar{\theta}$, then $\frac{\partial E(\tilde{p})}{\partial K} < 0$ and $\frac{\partial E(\tilde{v})}{\partial K} > 0$.

Figure 4 graphically illustrates Proposition 3. In the top two panels, we set $\bar{\theta} = 5$ and $\bar{c} = 1$, while in the bottom two panels, we set $\bar{\theta} = 1$ and $\bar{c} = 5$. In all panels, the other parameters are: $\tau_\varepsilon = 0.1, \tau_c = \tau_\theta = \alpha = \gamma = 1$, and $J = 20$. Consistent with Proposition 3, we observe that in Panel A1, there is a downward futures price bias and it declines with the number of the number K of financial traders. In Panel B1, there is an upward bias, and its absolute value also declines with K. In addition, in Panel A2 where $\bar{\theta} > \bar{c}$, the average spot price $E(\tilde{v})$ decreases with K, while the average futures price $E(\tilde{p})$ increases with K, while in Panel B2 where $\bar{\theta} < \bar{c}$, $E(\tilde{v})$ increases with K and $E(\tilde{p})$ decreases with K.

Panel B1 offers a possible explanation for the recent behavior of the crude oil market. Since June 2014, the crude oil price has kept declining from more than \$110 per barrel down to about \$30 per barrel in Feburary 2016, a more than 70% plunge. Many observers believe

participation of well-informed financial investors may enhance the quality of price signals."

that this falling price is predominantly a supply effect.[8] As we know, it takes time to search and develop oil fields. It is plausible that back to 2007–2008, commodity financialization has pushed oil futures price way too high, with its peak close to \$140 per barrel, and oil producers may have started to develop too many oil fields in which they would not invest otherwise. A few years later, these oil fields are fully developed and the new resulting oil wells generate excess oil supply, which is responsible for the price decline in the current oil market.

6 Welfare Implications

6.1 Welfare computations

We use the date-0 ex ante expected utility to represent the welfare of each group of agents. For consumers, we insert their date-1 wheat demand function (2) into their preference expression (1) and use the equilibrium spot price (10) to obtain consumers' date-1 indirect utility as follows:

$$\tilde{U}_{C,1} = \frac{1}{2} \left(\tilde{p} - \tilde{c} \right)^2 . \tag{26}$$

Note that $\frac{1}{2} \left(\tilde{p} - \tilde{c} \right)^2$ is also growers' equilibrium profits from producing wheat. So equation (26) says that after the spot market clears, consumers' welfare coincides with growers' profits. Taking expectation over $\tilde{U}_{C,1}$ yields consumers' date-0 expected utility (or certainty equivalent given that their preference is linear in money consumption and thus they are risk neutral) as follows:

$$CE_{C,0} \equiv E \left(\tilde{U}_{C,1} \right) = \frac{1}{2} \left[E \left(\tilde{p} - \tilde{c} \right) \right]^2 + \frac{1}{2} Var \left(\tilde{p} - \tilde{c} \right) . \tag{27}$$

For financial traders, we can compute their indirect utility after trading as

$$\widetilde{CE}_{F,1} = \frac{\left[E \left(\tilde{v} - \tilde{p} | \mathcal{I}_{F,k} \right) \right]^2}{2 \gamma Var \left(\tilde{v} | \mathcal{I}_{F,k} \right)}, \tag{28}$$

which is essentially the trading gains captured by financial traders in the futures market conditional on the realizations of the futures price $\tilde{p}$ and the private signal $\tilde{s}_k$. Using an argument similar to Grossman and Stiglitz (1980), we can show that the date-0 certainty

[8] For instance, in speaking to *Wall Street Journal*, Christine Lagarde, managing director of the International Monetary Fund, says: "What we do first is try to analyze whether it's a supply or demand effect. And in the present circumstances, it's predominantly supply. It's 80% supply, 20% demand." ("How the IMF's Christine Lagarde Sees the World's Economic Hot Spots," *Wall Street Journal*, 2014 December 9).

equivalent before the realizations of $\tilde{p}$ and $\tilde{s}_k$ is

$$\begin{aligned} CE_{F,0} &\equiv -\frac{1}{\gamma}\log\left[E\left(e^{-\gamma \widetilde{CE}_{F,1}}\right)\right] \\ &= \frac{\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{2\gamma Var\left(\tilde{v}-\tilde{p}\right)} + \frac{1}{2\gamma}\log\left[\frac{Var\left(\tilde{v}-\tilde{p}\right)}{Var\left(\tilde{v}|\mathcal{I}_{F,k}\right)}\right]. \end{aligned} \tag{29}$$

This expression is intuitive. The first term is the certainty equivalent that a trader can obtain without making demands dependent on the equilibrium price or any private information, where the numerator $\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2$ captures the potential gains due to the deviations between the futures contract's price $\tilde{p}$ and its payoff $\tilde{v}$ and the denominator $2\gamma Var\left(\tilde{v}-\tilde{p}\right)$ captures the risk in trading. The second term in (29) represents the additional benefit from trading with superior information, where $Var\left(\tilde{v}-\tilde{p}\right)$ is the benchmark risk when there is no information, while $Var\left(\tilde{v}|\mathcal{I}_{F,k}\right)$ is the reduced risk due to the additional information in the futures price $\tilde{p}$ and the private signal $\tilde{s}_k$.

Similarly, we can compute the indirect utility of growers after trading and production as follows:

$$\widetilde{CE}_{G,1} = \frac{\left[E\left(\tilde{v}-\tilde{p}|\mathcal{I}_G\right)\right]^2}{2\alpha Var\left(\tilde{v}|\mathcal{I}_G\right)} + \frac{1}{2}\left(\tilde{p}-\tilde{c}\right)^2, \tag{30}$$

where the first term captures the trading gains from participating in the futures market, while the second term is the profit from producing wheat (recall that growers take the futures price $\tilde{p}$ as the effective selling price of wheat). The date-0 ex ante certainty equivalent before the realizations of $\tilde{p}$ and $\tilde{c}$ is

$$\begin{aligned} CE_{G,0} &\equiv -\frac{1}{\alpha}\log\left[E\left(e^{-\alpha \widetilde{CE}_{G,1}}\right)\right] \\ &= \frac{1}{\alpha}\left(Q_0 - \frac{1}{2}Q_1'\left(I+2\Sigma Q_2\right)^{-1}\Sigma Q_1\right) + \frac{1}{2\alpha}\log\left|I+2\Sigma Q_2\right|, \end{aligned} \tag{31}$$

where I is the 2×2 identity matrix, Σ is covariance matrix of $\left(E\left(\tilde{v}-\tilde{p}|\mathcal{I}_G\right), \tilde{p}-\tilde{c}\right)'$, and

$$\begin{aligned} Q_2 &= \begin{bmatrix} \frac{\tau_\theta+K\tau_\varepsilon}{2} & 0 \\ 0 & \frac{\alpha}{2} \end{bmatrix}, Q_1 = \begin{bmatrix} \left(\tau_\theta+K\tau_\varepsilon\right)E\left(\tilde{v}-\tilde{p}\right) \\ \alpha E\left(\tilde{p}-\tilde{c}\right) \end{bmatrix}, \\ \text{and } Q_0 &= \frac{\left(\tau_\theta+K\tau_\varepsilon\right)\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2}{2} + \frac{\alpha}{2}\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2. \end{aligned}$$

Again, in (31), the first term captures the certainty equivalent that a grower can obtain without making decisions based on private information $\tilde{c}$ and the futures price $\tilde{p}$, while the second term relates to the extra benefit from speculating on superior information.

6.2 Symmetric information economy: The role of risk sharing

We now set $\tau_\varepsilon = 0$, so that the presence of more financial traders in the futures market simply adds more market participants without adding new information to the market. In this case, financial traders can infer the private information $\tilde{c}$ owned by growers, so that the futures market features symmetric information. Since no information on $\tilde{\theta}$ is brought into the market, the variations in the futures price $\tilde{p}$ are fully driven by the variations in the cost $\tilde{c}$ of the wheat production technology. In this limiting economy, we can compute the welfare expressions analytically, which are given in the appendix.

Consumers' welfare increases with the number K of financial traders. Note that by equation (26), after the spot market clears, consumers' welfare coincides with growers' profits. When there are more financial traders in the futures market, growers can hedge better their wheat production, which effectively improves their profit. Specifically, recall that the futures price $\tilde{p}$ is also the effective selling price of wheat in growers' production decision. When the cost $\tilde{c}$ is low, growers tend to produce more, but by Proposition 1), $\tilde{p}$ tends to be low as well (i.e., $p_c = \frac{\partial \tilde{p}}{\partial \tilde{c}} > 0$), which lowers growers' revenue. As more financial traders come to the futures market, their trading causes the price to be less senstive to $\tilde{c}$ (i.e., $\frac{\partial p_c}{\partial K} < 0$). This reduced price impact of $\tilde{c}$ increases growers' profits and hence consumers' welfare by (26).

The welfare of each financial trader decreases with the number K of financial traders in the futures market for two reasons. First, as more financial traders participate in the market, they bring down the futures price bias $\left[E\left(\tilde{v}-\tilde{p}\right)\right]^2$ by Proposition 3, and thus, the first term in (29), which corresponds to the trading gains without information, will decrease. Second, the second term in (29) also decreases with K, because more financial traders, who have the same information as growers, also bring the price $\tilde{p}$ closer to its payoff $\tilde{v}$, which effectively reduces $Var\left(\tilde{v}-\tilde{p}\right)$.

The effect of commodity financialization on the welfare of growers is ambiguous: increasing K will benefit growers if and only if the number of financial traders is sufficiently large. This is in contrast to our conventional wisdom that expanding the traders base would benefit growers (as hedgers) through more risk sharing in the market. We can demonstrate the intuition most clearly by examining equation (30), which is the indirect utility $\widetilde{CE}_{G,1}$ post trading and production. We also set $\tau_c = \infty$ so that $\tilde{c} = \bar{c}$. By so doing, we essentially remove the randomness in the futures price $\tilde{p}$, and thus $\widetilde{CE}_{G,1} = CE_{G,0}$. We can show that increasing K affects the two terms of (30), $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$ and $\frac{1}{2}(\tilde{p}-\tilde{c})^2$, in opposite directions: it increases $\frac{1}{2}(\tilde{p}-\tilde{c})^2$ but decreases $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$. First, when there are more financial traders, they can share the risk loaded off by growers, thereby increasing the profit $\frac{1}{2}(\tilde{p}-\tilde{c})^2$ made by growers (see the discussion on consumers' welfare). Second, the presence of more financial traders also lowers the futures price bias, which therefore reduces the trading gains $\frac{[E(\tilde{v}-\tilde{p}|\mathcal{I}_G)]^2}{2\alpha Var(\tilde{v}|\mathcal{I}_G)}$. This second negative effect dominates when there are not many financial traders in the market and so the futures bias is initially large.

Interestingly, we can also show that the dominance of the negative effect on growers' welfare only arises in our production economy in which growers make real investment decisions. In contrast, in an endowment economy in which growers are given with an exogenous amount of commodities, increasing the number of financial traders always benefits growers. That is, the negative effect is stronger in our production economy. This is because when the futures price $\tilde{p}$ increases due to the added financial traders, growers also supply more wheat in the later spot price, which therefore also endogenously changes the payoff on the futures contract, making the decrease in the futures price bias particularly severe.

Proposition 4 *Suppose $\tau_\varepsilon = 0$ so that the futures market features symmetric information.*
(a) Increasing the number K of financial traders benefits consumers and harms financial traders. That is, $\frac{\partial CE_{C,0}}{\partial K} > 0$ and $\frac{\partial CE_{F,0}}{\partial K} < 0$.
(b) If, in addition $\tau_c = \infty$ and $\bar{c} \neq \bar{\theta}$, growers' welfare improves with financialization if and only if $\frac{K}{\gamma} > \frac{J}{\alpha}$. That is, if $\tau_c = \infty$ and $\bar{c} \neq \bar{\theta}$, then $\frac{\partial CE_{G,0}}{\partial K} > 0$ if and only if $\frac{K}{\gamma} > \frac{J}{\alpha}$.

6.3 Asymmetric information economy: The role of price discovery

Now we allow $\tau_\varepsilon > 0$, so that the futures market participation of financial traders also brings new information about the demand shock $\tilde{\theta}$ into the market. We find that this new feature mainly changes the implication for growers' welfare. Because of the complexity of the welfare expressions, it is difficult to establish analytical results. We therefore use a numerical example to illustrate our analysis. We have tried numerous parameter configurations and found that the pattern we identified is quite robust.

In Figure 5, we choose parameter values similar to those in previous figures. That is, $\tau_\varepsilon = 0.1$, $\tau_c = \tau_\theta = \alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$, and $\bar{c} = 1$. The interesting observation concerns the welfare of growers in Panel A. Recall that in Proposition 4, when $\tau_\varepsilon = 0$, growers' welfare $CE_{G,0}$ first decreases and then increases with K. Now when $\tau_\varepsilon > 0$, we find that $CE_{G,0}$ first decreases, then increases, but finally decreases again with K. This suggests that the negative welfare effect on the trading gains is particularly strong either when K is sufficiently small or when K is sufficiently large. The intuition for the case of small K is still the same as before, that is, when there are not many financial traders in the market, the futures bias can be large, leaving a large room for it to decline. Now, when $\tau_\varepsilon > 0$, recall that financial traders bring information to the market. Thus, when there are many financial traders who infer information from prices, a newly added financial trader will cause all these traders to reform their forecast, and after aggregating their trading, the price can reveal a lot more information. This price discovery process will hurt the growers' trading gains through a channel similar to the Hirshleifer effect (1971). That is, more financial traders bring more information about the payoff on the futures payoff, thereby destroying the potential trading gains that can be captured by market participants.

Panels B and C of Figure 5 suggest that commodity financialization still benefits consumers and harms financial traders. The aforementioned price discovery effect also adversely affects financial traders. However, since when $\tau_\varepsilon = 0$, the welfare of each financial trader has already declined with K, the extra negative effect due to price discovery only strengthens this pattern and will not change the direction. Note that here it is the welfare of each individual financial trader that decreases with K. As a group, financial traders' aggregate welfare $K \times CE_{F,0}$ actually first increases and then decreases with K.

7 The Feedback Effect, Market Efficiency, and Welfare

In the model presented in Section 3, growers make production decisions after observing the futures price $\tilde{p}$, which establishes a feedback effect of futures price on the later spot price through the supply channel (i.e., Lemma 1). In this section, we consider an extension with two types of growers to examine the role of this feedback effect in determining market efficiency and agents' welfare.

7.1 The setup and equilibrium of the extended economy

Let us divide the J growers into two groups: Group A which includes μJ growers (with $\mu \in [0,1]$) and Group B which includes $(1-\mu) J$ growers. Growers in Group A behave in the same way as in our baseline model—i.e., they have access to the futures price $\tilde{p}$ when they make production decisions x_A.[9] By contrast, growers in Group B have to make production decisions x_B before the futures market clears, and thus the wheat supply from this group of growers does not depend on the futures price $\tilde{p}$. That is, the futures price $\tilde{p}$ affects the later spot price $\tilde{v}$ only through the supply of A-growers. In this way, the fraction μ of A-growers controls the strength of the feedback effect of $\tilde{p}$ on $\tilde{v}$.

The other features remain the same as in Section 3. Specifically, at date 0, all growers' productions incur a cost according to equation (3) and they know $\tilde{c}$ when making production decisions. Growers trade futures contracts to maximize preference given by equation (4). Financial trader k observes the private signal $\tilde{s}_k$ and trades futures contracts to maximize (7). At date 1, consumers observe the demand shock $\tilde{\theta}$, choose wheat consumption to maximize (1), which forms the wheat demand function in the spot market .

In this extended economy, the futures price function still takes the form of (11). However, the spot price $\tilde{v}$ will be different from equation (10), as B-growers' wheat supply function differs from that of A-growers. Nonetheless, $\tilde{v}$ is still normally distributed. The decision problems of consumers, financial traders, and A-growers are the same as in the baseline model. That is, consumers' wheat demand function $y\left(\tilde{\theta}, \tilde{v}\right)$ is given by equation (2). Financial trader k's futures demand function $d_F\left(\tilde{s}_k, \tilde{p}\right)$ is given by (15). The wheat supply $x_A\left(\tilde{c}, \tilde{p}\right)$ from A-growers is given by equation (9), while their futures demand $d_A\left(\tilde{c}, \tilde{p}\right)$ is given by equation (12). Note that the expressions of conditional moments in (12) and (15)—i.e., $E\left(\tilde{v}|\mathcal{I}_G\right)$, $Var\left(\tilde{v}|\mathcal{I}_G\right)$, $E\left(\tilde{v}|\mathcal{I}_{F,k}\right)$, and $Var\left(\tilde{v}|\mathcal{I}_{F,k}\right)$—need to be recomputed appropriately because the equilibrium spot price $\tilde{v}$ will be different in the presence of B-growers.

Now let us examine the decision problems of the new type of agents, B-growers. They

[9] So, the letter A refers to "*a*djustable" commodity supply after the futures market clears.

make two *sequential* decisions at date 0: first, they decide on the wheat quantity x_B to produce given private information $\{\tilde{c}\}$, and second, they then decide on the quantity d_B of futures contracts to hold in the futures market given information $\{\tilde{c}, \tilde{p}\}$. We work out these problems backward by first solving the futures investment problem.

In the futures market, B-growers take the wheat production x_B as given and choose futures investment d_B (and investment in the risk-free asset) to maximize

$$E\left(-e^{-\alpha\left[\tilde{v}x_B - \tilde{c}x_B - \frac{1}{2}x_B^2 + (\tilde{v}-\tilde{p})d_B\right]}\middle|\, \tilde{c}, \tilde{p}\right). \tag{32}$$

Solving the above problem delivers

$$d_B\left(\tilde{c}, \tilde{p}\right) = \underbrace{\frac{E\left(\tilde{v}|\tilde{c}, \tilde{p}\right) - \tilde{p}}{\alpha Var\left(\tilde{v}|\tilde{c}, \tilde{p}\right)}}_{\text{speculation}} - \underbrace{x_B}_{\text{hedging}}. \tag{33}$$

Comparing (33) with (12), we find that B-growers' demand for futures differ from that of A-growers only to the extent of their optimal wheat production that needs to be hedged.

Inserting the futures demand $d_B\left(\tilde{c}, \tilde{p}\right)$ into the objective function (32) yields the indirect value function of B-growers at the futures market. Taking expectation with respect to the futures price $\tilde{p}$ conditional on $\tilde{c}$ delivers the objective function of B-growers when they choose the optimal wheat production, x_B. In the appendix, we show that the optimal wheat production x_B linearly depends on $\tilde{c}$, i.e.,

$$x_B = b_0 - b_c\tilde{c}, \tag{34}$$

where b_0 and b_c are endogenous constants.

The futures market clearing condition is

$$\mu J \times d_A\left(\tilde{c}, \tilde{p}\right) + \left(1-\mu\right) J \times d_B\left(\tilde{c}, \tilde{p}\right) + \sum\nolimits_{k=1}^{K} d_F\left(\tilde{s}_k, \tilde{p}\right) = 0. \tag{35}$$

This equation, together with the expressions of the demand functions, determines the futures price function.

At date 1, the spot market clearing condition is

$$\mu J \times x_A\left(\tilde{c}, \tilde{p}\right) + \left(1-\mu\right) J \times x_B\left(\tilde{c}\right) = J \times y\left(\tilde{\theta}, \tilde{v}\right), \tag{36}$$

where the left-hand side is the total wheat supply from A-growers and B-growers, while the right-hand side is the aggregate wheat demand from consumers. Using (2), (9), and (34), we can compute

$$\tilde{v} = \tilde{\theta} + v_c\tilde{c} - \mu\tilde{p} - v_0, \tag{37}$$

where

$$v_c \equiv \mu + (1-\mu)\, b_c \text{ and } v_0 \equiv (1-\mu)\, b_0. \tag{38}$$

Equation (37) verifies that in equilibrium, the spot price $\tilde{v}$ is indeed normally distributed.

In the appendix, we prove the following characterization proposition.

Proposition 5 *In the linear REE of the extended economy, the futures price $\tilde{p}$ and the spot price $\tilde{v}$ take the form of*

$$\begin{aligned} \tilde{p} &= p_0 + p_s \tilde{S} + p_c \tilde{c}, \\ \tilde{v} &= \tilde{\theta} + v_c \tilde{c} - \mu \tilde{p} - v_0, \end{aligned}$$

with $v_c \equiv \mu + (1-\mu)\, b_c$ and $v_0 \equiv (1-\mu)\, b_0$, where $p_0, p_s, p_c, b_0,$ and b_c are determined by the following system

$$p_0 = \frac{J\left[\frac{\tau_\theta \bar{\theta} - (\tau_\theta + K\tau_\varepsilon) v_0}{\alpha} - (1-\mu)\, b_0\right] + K \frac{\left(1 - \rho\beta_{F,p} - \beta_{F,s}\right)\bar{\theta} + \left(v_c - \beta_{F,p}\right)\bar{c} - v_0}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}{J\left[\frac{(\tau_\theta + K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right] + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}, \tag{39}$$

$$p_s = \frac{J\frac{K\tau_\varepsilon}{\alpha} + K\frac{\rho\beta_{F,p} + \beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}{J\left[\frac{(\tau_\theta + K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right] + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}, \tag{40}$$

$$p_c = \frac{J\left(\frac{(\tau_\theta + K\tau_\varepsilon) v_c}{\alpha} + [\mu + (1-\mu)\, b_c]\right) + K\frac{\beta_{F,p}}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}{J\left[\frac{(\tau_\theta + K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right] + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}}, \tag{41}$$

$$b_0 = \frac{p_0 + p_s\bar{\theta} - \frac{h_0 Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}{1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}, \tag{42}$$

$$b_c = \frac{-p_c + 1 + \frac{h_c Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}{1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}, \tag{43}$$

where

$$\begin{aligned} \rho &= \frac{p_s}{p_c}, \\ \beta_{F,p} &= K\tau_\varepsilon \frac{\tau_c (K-1)\rho + K v_c (\tau_\theta + \tau_\varepsilon)}{\tau_c (K-1)(\tau_\theta + K\tau_\varepsilon)\rho^2 + K^2\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)}, \\ \beta_{F,s} &= K\tau_\varepsilon \frac{-v_c(\tau_\theta + K\tau_\varepsilon)\rho + K\tau_\varepsilon}{\tau_c (K-1)(\tau_\theta + K\tau_\varepsilon)\rho^2 + K^2\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)}, \\ Var\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right) &= \frac{(K-1)\left(\tau_c + \tau_\theta v_c^2 + K\tau_\varepsilon v_c^2\right)\rho^2 - 2K\tau_\varepsilon v_c (K-1)\rho + K^2\tau_\varepsilon}{\tau_c (K-1)(\tau_\theta + K\tau_\varepsilon)\rho^2 + K^2\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)}, \end{aligned}$$

$$
\begin{aligned}
Var\left(\tilde{p}|\tilde{c}\right) &= p_s^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right), \\
Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right) &= \frac{1}{\tau_\theta+K\tau_\varepsilon}+\left(\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}-(\mu+1)\,p_s\right)^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right), \\
Cov\left(\tilde{p},E\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c}\right)|\tilde{c}\right) &= p_s\left(\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}-(\mu+1)\,p_s\right)\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right), \\
h_0 &= \left[1-(\mu+1)\,p_s\right]\bar{\theta}-v_0-(\mu+1)\,p_0, \\
h_c &= v_c-(\mu+1)\,p_c.
\end{aligned}
$$

Given the complexity of the system, our subsequent analysis relies on numerical analysis. To compute the equilibrium, we first use equations (40), (41), and (43) to solve the three unknowns p_s, p_c, and b_c. After we obtain these three unknowns, we then use equations (39) and (42) to form a linear system in terms of p_0 and b_0.

7.2 Implications for market efficiency and welfare

7.2.1 The feedback effect and market efficiency

Market efficiency, also labeled as price efficiency and informational efficiency, refers to the extent to which the prevailing market price reflects the future value of the traded asset. In our case, the traded asset is the futures contract, and its price and cash flow are $\tilde{p}$ and $\tilde{v}$, respectively. The correlation coefficient $Corr\left(\tilde{v},\tilde{p}\right)$ between the futures and spot prices is a measure for market efficiency. Regulators and academics often view promoting market efficiency as one desirable goal. The idea is that an informationally efficient market can effectively guide resource allocation—i.e., a high market price of an asset indicates that the underlying business of the asset is in high demand, which in turn attracts more resources and benefits the entire society (Fama and Miller, 1972). We use Figures 6 and 7 to respectively examine the implications of the feedback effect for market efficiency and welfare.

In Figure 6, we plot $Corr\left(\tilde{v},\tilde{p}\right)$ against the fraction μ of A-growers. The other parameter values are: $\tau_\varepsilon=0.1$, $\tau_c=\tau_\theta=\alpha=\gamma=\bar{\theta}=\bar{c}=1$ and $J=K=20$.[10] We find that $Corr\left(\tilde{v},\tilde{p}\right)$ monotonically decreases with μ and thus in our setting, the feedback effect weakens market efficiency. The intuition lies in equation (37). As μ increases, the A-group

[10] The result in Figure 6 is robust to the choice of parameter values.

has more growers whose wheat supply depends positively on the futures price $\tilde{p}$. So, seeing a high $\tilde{p}$, these A-growers will supply a lot more wheat in the spot market, driving down the spot price $\tilde{v}$, and vice versa. Through this supply channel, an increase in μ tends to reduce the correlation between $\tilde{p}$ and $\tilde{v}$.

The result presented in Figure 6 differs from those in other feedback effect settings. For instance, suppose that the traded asset is the stock on a firm. The firm manager can learn from its stock price to guide real decisions, which establishes a feedback effect from the stock price to its cash flows. More specifically, the firm may be uncertain about the future demand for its product, which depends on consumers' preference. The stock price incorporates such demand side information through consumers' investment in the firm's stocks, so that the firm can learn from its own stock price. In this alternative setting, the feedback effect tends to strengthen market efficiency: a high stock price signifies a strong demand for the firm's product, which in turn guides the firm to make more informed decisions, thereby indeed improving its fundamentals.

Figure 6 also makes new empirical predictions on market efficiency based on the strength of the feedback effect driven by the supply channel. For instance, for a fixed horizon, say, one year, the agriculture industry is far more flexible in adjusting supply than the oil industry. We therefore expect that the feedback effect is stronger for agricultural firms than for oil firms. Thus, according to Figure 6, other things being equal, the stock prices of agricultural companies should be less informationally efficient than the stock prices of oil firms.

7.2.2 Feedback effect and welfare

We use Figure 7 to plot various welfare variables against μ for the same parameter value in Figure 6. Variables CE_A, CE_B, CE_C, and CE_F represent the date-0 certainty equivalents before realizations of any random variables for A-growers, B-growers, consumers, and financial traders, respectively (this is similar to equations (27), (29), and (31) in the baseline model). The exact expressions are given in the appendix. In Panel C, we also report the weighted average of CE_A and CE_B,

$$CE_G \equiv \mu CE_A + (1-\mu)\, CE_B,$$

as a proxy for the welfare of an average grower without knowing the type. In Panel F, we report an aggregate welfare measure, which is the sum of the certainty equivalents of all agents, i.e.,

$$Agg.Wel. \equiv \mu J \times CE_A + (1-\mu) J \times CE_B + J \times CE_C + K \times CE_F.$$

We find that the reduced informational efficiency (due to the feedback effect) in Figure 6 does not necessarily translate into a lower welfare in Figure 7, which highlights the delicate link between informational efficiency and welfare. Specifically, we compare two extreme economies: (1) $\mu = 0$ vs. (2) $\mu = 1$. In the first economy, all growers are in the B-group, so that the feedback effect is shut down. In the second economy, all growers are A-growers, and thus the feedback effect is the strongest. Panels C, D, and E reveal that for the parameter configuration in Figure 7, all agents—growers, consumers, and financial traders—are better off in the second economy with the feedback effect. Of course, in Panel F, the aggregate welfare in the second economy is higher as well.

The intuition for this welfare result is as follows. In the second economy with only A-growers, growers can make more informed investment decisions by observing the futures price $\tilde{p}$ that conveys information about the wheat demand shock $\tilde{\theta}$ in the later spot market, and thus their welfare increases from this more informed production decision. This is simply Blackwell's (1951) theorem that the ex ante expected utility of a single decision maker under a finer information set is weakly higher than under a coarser information set. Panel C of Figure 7 suggests that the partial equilibrium intuition of Blackwell's (1951) theorem can hold in our general equilibrium setting.

Consumers also partly enjoy the benefit of the more informed productions because they are now served better by commodity producers. For financial traders, their benefit comes from trading against more uninformed trading in the second economy. Specifically, since A-growers can adjust their production when trading the futures contracts, their production is more responsive to the cost shock $\tilde{c}$ than the production of B-growers (more formally, $b_c < 1$), which in turn means that A-growers hedge more in the futures market than B-growers. As a result, there is more hedging-motivated trading in the futures market when the economy is populated by A-growers only than by B-growers only. This benefits financial traders who speculate in the financial market and gain at the expense of uninformed trading.

8 Conclusion

In the past decade, there is a sharp increase in the inflow of financial investors into commodity futures markets, which is labelled as the financialization of commodities. In this paper, we develop a model to study the implications of this phenomenon for trading behavior, asset prices, and welfare through the lens of risk sharing and price discovery. Our analysis highlights a supply channel through which the futures price affects the commodity spot price, which establishes the real effect of financial activities in the commodity futures market. Financial traders as speculators can either provide liquidity to or demand liquidity from other futures market participants such as commercial hedgers, depending on the information environment. Commodity financialization helps to reduce futures price bias, not only because financial traders help to share risk, but also because they bring new information to the market, which reduces the risk faced by all market participants. Commodity financialization generally harms financial traders and benefits the final end consumers. Unlike the conventional wisdom that argues that commercial hedgers benefit from the presence of more market participants, commercial hedgers can actually lose in the process of commodity financialization, because more financial traders active in the futures market also reduces the trading gains of commercial hedgers through bringing down the futures price bias.

Appendix

Proof of Proposition 1

In order to get equation (22) determining ρ, we first plug all the conditional moments into the demand functions of each type of traders, then plug the expressed demand function into the market clearing condition, to write the equilibrium price $\tilde{p}$ as functions of $\left(\tilde{c}, \tilde{S}\right)$, and finally compare compare coefficients. Specifically, in the aggregate order flow, information about $\tilde{c}$ is brought by growers, and $\tilde{S}$ is brought by growers and financial traders. We compute the coefficient on $\tilde{c}$ in the aggregate order flow as $J\left[\frac{1}{\alpha Var\left(\tilde{\theta}|\tilde{S}\right)}+1\right]$ and the coefficient on $\tilde{S}$ is $J\frac{K\tau_{\varepsilon}}{\alpha}+K\frac{\beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}$. Thus, we have:

$$\rho = \frac{p_s}{p_c} = \frac{J\frac{K\tau_{\varepsilon}}{\alpha}+K\frac{\beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left[\frac{1}{\alpha Var\left(\tilde{\theta}|\tilde{S}\right)}+1\right]}. \tag{A1}$$

Plugging the expressions of $\beta_{F,s}$ and $Var\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right)$ into the above expression yields (22).

Examining equation (22), we find that when $\rho = 0$, the RHS is positive, and that when $\rho \to \infty$, the RHS is finite. Thus, by the intermediate value theorem, there exists a solution $\rho \in (0, \infty)$ to equation (22). In effect, we can further narrow down the range of ρ as $\left[\frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}+\alpha}, \frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}}\right]$. To see this, suppose $\rho > \frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}}$, so that $\beta_{F,s} < 0$. Then, by equation (22), we must have $\rho \leq \frac{J\frac{K\tau_{\varepsilon}}{\alpha}}{J\left(\frac{1}{\alpha Var\left(\tilde{\theta}|\tilde{S}\right)}+1\right)} = \frac{\frac{K\tau_{\varepsilon}}{\alpha}}{\frac{\tau_{\theta}+K\tau_{\varepsilon}}{\alpha}+1} = \frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}+\alpha}$. A contradiction. Thus, we must have $\rho \leq \frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}}$ and $\beta_{F,s} \geq 0$. Accordingly, by equation (22), we have $\rho \geq \frac{J\frac{K\tau_{\varepsilon}}{\alpha}}{J\left(\frac{1}{\alpha Var(\tilde{\theta}|\tilde{S})}+1\right)} = \frac{K\tau_{\varepsilon}}{\tau_{\theta}+K\tau_{\varepsilon}+\alpha}$.

Next, we establish the uniqueness of the equilibrium when α is sufficiently small. Note that only $\frac{\beta_{F,s}}{Var(\tilde{v}|\tilde{s}_k,\tilde{p})}$ depends on ρ in the RHS of (A1). By the expressions of $\beta_{F,s}$ and $Var\left(\tilde{v}|\tilde{s}_k, \tilde{p}\right)$, we have

$$\begin{aligned}
&\frac{\partial}{\partial\rho}\frac{\beta_{F,s}}{Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)} \\
= \; &\frac{\partial}{\partial\rho}\frac{K\tau_{\varepsilon}\left(-\left(\tau_{\theta}+K\tau_{\varepsilon}\right)\rho+K\tau_{\varepsilon}\right)}{(K-1)\left(\tau_c+\tau_{\theta}+K\tau_{\varepsilon}\right)\rho^2-2K\tau_{\varepsilon}(K-1)\rho+K^2\tau_{\varepsilon}} \\
= \; &K\tau_{\varepsilon}\frac{\left[\begin{array}{c} -\left(\tau_{\theta}+K\tau_{\varepsilon}\right)\left((K-1)\left(\tau_c+\tau_{\theta}+K\tau_{\varepsilon}\right)\rho^2-2K\tau_{\varepsilon}(K-1)\rho+K^2\tau_{\varepsilon}\right) \\ -\left(-\left(\tau_{\theta}+K\tau_{\varepsilon}\right)\rho+K\tau_{\varepsilon}\right)\left(2(K-1)\left(\tau_c+\tau_{\theta}+K\tau_{\varepsilon}\right)\rho-2K\tau_{\varepsilon}(K-1)\right)\end{array}\right]}{\left((K-1)\left(\tau_c+\tau_{\theta}+K\tau_{\varepsilon}\right)\rho^2-2K\tau_{\varepsilon}(K-1)\rho+K^2\tau_{\varepsilon}\right)^2}.
\end{aligned}$$

The numerator of the above expression is quadratic and it is downward sloping for $\rho \leq \frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon}$. In addition, this quadratic numerator is negative at $\rho = \frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon}$. Note that in equilibrium, $\rho \in \left(\frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon + \alpha}, \frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon}\right)$, and thus, when α is sufficiently small, $\frac{\partial}{\partial \rho}\frac{\beta_{F,s}}{Var(\tilde{v}|\tilde{s}_k,\tilde{p})} < 0$ for all $\rho \in \left[\frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon + \alpha}, \frac{K\tau_\varepsilon}{\tau_\theta + K\tau_\varepsilon}\right]$. As a result, the RHS of (A1) is downward sloping in ρ, while its RHS is upward sloping. Therefore, uniqueness is established.

Finally, using the market clearing condition, we can compute the expressions of p_0, p_s and p_c as follows:

$$
\begin{aligned}
p_0 &= \frac{J\frac{\tau_\theta \bar{\theta}}{\alpha} + K\frac{\left(1-\rho\beta_{F,p}-\beta_{F,s}\right)\bar{\theta}+\left(1-\beta_{F,p}\right)\bar{c}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}, \\
p_s &= \frac{J\frac{K\tau_\varepsilon}{\alpha} + \frac{K\left(\beta_{F,p}\rho+\beta_{F,s}\right)}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}, \\
p_c &= \frac{J\left(\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+1\right) + \frac{K\beta_{F,p}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}.
\end{aligned}
$$

Note that $p_s \geq 0$ and $p_c > 0$ because $J > 0$, $K \geq 0$ and both $\beta_{F,p}$ and $\beta_{F,s}$ are non-negative.

Proof of Proposition 2

We prove $Corr\left(d_F, \tilde{p}\right) < 0$ for small values of τ_ε or τ_c by considering two limiting economies. First, for any given $\tau_c \in [0, \infty)$, when $\tau_\varepsilon \to 0$, we have $Corr\left(d_F, \tilde{p}\right) \to -1$. To see this, by setting $\tau_\varepsilon = 0$, we can use Proposition 1 to show

$$
\tilde{p} = \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\bar{\theta} + \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\tilde{c},
$$

$$
d_F\left(\tilde{s}_k, \tilde{p}\right) = \frac{E\left(\tilde{v}-\tilde{p}|\tilde{s}_k,\tilde{p}\right)}{\gamma Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right)} \propto \tilde{p}-\tilde{c} = \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\left(\bar{\theta}-\tilde{c}\right).
$$

Thus, as long as $Var\left(\tilde{c}\right) > 0$, we have $Corr\left(\tilde{p}-\tilde{c}, \tilde{p}\right) = -1$. Second, for any given $\tau_\varepsilon > 0$, when $\tau_c \to 0$, we also have $Corr\left(d_F, \tilde{p}\right) \to -1$. Note that when $\tau_c \to 0$, we have $Var\left(\tilde{c}\right) \to \infty$, and thus the variations in $d_F\left(\tilde{s}_k, \tilde{p}\right)$ and $\tilde{p}$ are primarily driven by variations in $\tilde{c}$. Again, by Proposition 1, we can show that as long as $\tau_\varepsilon > 0$, the coefficient p_c on $\tilde{c}$ in the price $\tilde{p}$ is given by

$$
p_c = \frac{J\left(\frac{\tau_\theta+K\tau_\varepsilon}{\alpha}+1\right)+\frac{K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{2(\tau_\theta+K\tau_\varepsilon)}{\alpha}+1\right)+\frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}} \in (0,1)
$$

and the coefficient of $\tilde{c}$ in $d_F(\tilde{s}_k, \tilde{p})$, it is proportional to

$$1 - 2p_c = -\frac{J}{J\left(\frac{2(\tau_\theta + K\tau_\varepsilon)}{\alpha} + 1\right) + \frac{2K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}} < 0.$$

Thus, $Corr(d_F, \tilde{p}) \to -1$ as $\tau_c \to 0$ for a given $\tau_\varepsilon > 0$.

We prove $Corr(d_F, \tilde{p}) > 0$ for large values of τ_c by considering a limiting economy of $\tau_c \to \infty$. Setting $\tau_c \to \infty$ in Proposition 1, we can show

$$\tilde{p} = \frac{\left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)\tau_\theta \bar{\theta} + \left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)(\tau_\theta + K\tau_\varepsilon)\bar{c} + J\bar{c} + \left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)K\tau_\varepsilon \tilde{S}}{J + 2\left(\frac{J}{\alpha} + \frac{K}{\gamma}\right)(\tau_\theta + K\tau_\varepsilon)},$$

and $d_F \propto \tilde{p} - \bar{c}$. As a result, we have $Corr(d_F, \tilde{p}) = 1$.

Proof of Proposition 3

By the demand functions (12) and (15) and the market clearing condition (21), we can show

$$\bar{v} - \bar{p} = \frac{J(\bar{p} - \bar{c})}{\frac{J}{\alpha Var(\tilde{\theta}|\tilde{S})} + \frac{K}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}} = \frac{\bar{p} - \bar{c}}{A}, \tag{A2}$$

where A is given by (24) and $\bar{v} = E(\tilde{v})$ and $\bar{p} = E(\tilde{p})$. Then, we use the expression of $\tilde{v}$ in (10) to obtain

$$\bar{v} = \bar{\theta} + \bar{c} - \bar{p}. \tag{A3}$$

From equations (A2) and (A3), we can solve

$$\begin{aligned} \bar{p} &= \bar{c} + \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right), \\ \bar{v} &= \bar{\theta} - \frac{A}{2A+1}\left(\bar{\theta} - \bar{c}\right), \\ \bar{v} - \bar{p} &= \frac{\bar{\theta} - \bar{c}}{2A+1}. \end{aligned}$$

Clearly, $\bar{v} - \bar{p} > 0$ if and only if $\bar{\theta} > \bar{c}$.

Also, K affects $\bar{p}$ and $\bar{v}$ through affecting A. Given the expressions of $Var\left(\tilde{\theta}\middle|\tilde{S}\right)$ and $Var(\tilde{v}|\tilde{s}_k, \tilde{p})$ in (14) and (17), we can express A as follows:

$$A = \frac{\tau_\theta + K\tau_\varepsilon}{\alpha} + \frac{K}{J}\frac{\tau_c(K-1)(\tau_\theta + K\tau_\varepsilon)\rho^2 + K^2\tau_\varepsilon(\tau_\theta + \tau_\varepsilon)}{\gamma\left[(K-1)(\tau_c + \tau_\theta + K\tau_\varepsilon)\rho^2 - 2K\tau_\varepsilon(K-1)\rho + K^2\tau_\varepsilon\right]}.$$

Suppose $\tau_\varepsilon > 0$. Then, when α is small, the derivative $\frac{\partial A}{\partial K}$ is mainly driven by the first term, which increases with K. That is, $\frac{\partial A}{\partial K} \approx \frac{\tau_\varepsilon}{\alpha} > 0$. This effect occurs through the information effect on growers' trading. If $\tau_\varepsilon = 0$, then,

$$A = \frac{\tau_\theta}{\alpha} + \frac{K}{J}\frac{\tau_\theta}{\gamma},$$

and still, increasing K will increase A directly. This effect occurs through the effect of expanding the traders population. Therefore, independent of the value of τ_ε, A increases with K for small levels of α. All the other results follow immediately.

Proof of Proposition 4

By Proposition 1, we can show that when $\tau_\varepsilon = 0$, we have

$$\begin{aligned}
\tilde{p} &= \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\bar{\theta} + \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\tilde{c}, \\
\tilde{p}-\tilde{c} &= \frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\left(\bar{\theta}-\tilde{c}\right).
\end{aligned}$$

Thus, we can compute

$$Var\left(\tilde{p}-\tilde{c}\right) = \left[\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right]^2\frac{1}{\tau_c}, \tag{A4}$$

$$\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2 = \left[\frac{\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right]^2\left(\bar{\theta}-\bar{c}\right)^2. \tag{A5}$$

It is clear that both $\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2$ and $Var\left(\tilde{p}-\tilde{c}\right)$ increase with K.

Given the expression of $CE_{C,0}$ in (27), we know that $\frac{\partial CE_{C,0}}{\partial K} > 0$.

We can also show that

$$\widetilde{CE}_{F,1} = \frac{1}{2\gamma\tau_\theta}\left(\tilde{p}-\tilde{c}\right)^2\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2 \Rightarrow$$

$$\begin{aligned}
CE_{F,0} &= \frac{1}{2\gamma}\log\left[1+2\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\frac{1}{\tau_c}\right] \\
&\quad +\frac{1}{\gamma}\frac{\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\left(\bar{\theta}-\bar{c}\right)^2}{1+2\left(\frac{J\tau_\theta}{\sqrt{2\tau_\theta}}\frac{1}{2\tau_\theta\left(\frac{J}{\alpha}+\frac{K}{\gamma}\right)+J}\right)^2\frac{1}{\tau_c}}.
\end{aligned}$$

As K increases, both the first term and the second term of $CE_{F,0}$ decrease. Thus, $CE_{F,0}$ decreases with K.

For growers, we can show

$$\widetilde{CE}_{G,1} = \frac{1}{2}\left(\tilde{p}-\tilde{c}\right)^2\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right] \Rightarrow$$

$$
\begin{aligned}
CE_{G,0} &= \frac{1}{2\alpha}\log\left[1+\alpha\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]Var\left(\tilde{p}-\tilde{c}\right)\right] \\
&\quad +\frac{1}{2}\frac{\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]}{1+\alpha\left[\frac{1}{\alpha\tau_\theta}\left(\frac{J}{\frac{J}{\alpha}+\frac{K}{\gamma}}\right)^2+1\right]Var\left(\tilde{p}-\tilde{c}\right)}\left[E\left(\tilde{p}-\tilde{c}\right)\right]^2.
\end{aligned}
$$

Now suppose $\tau_c=\infty$ and $\bar{\theta}\neq\bar{c}$. Then, the first term in $CE_{G,0}$ vanishes by (A4), and then combined with (A5), direct computation shows $\frac{\partial CE_{G,0}}{\partial K}>0 \Leftrightarrow \frac{K}{\gamma}>\frac{J}{\alpha}$.

Proof of Proposition 5

We first derive the conditions determining p_0, p_s, and p_c, by examining the futures market. Growers have information set $\{\tilde{c},\tilde{p}\}$. Thus, they can back out signal $\tilde{S}$ from the price function (11). Using the expression of $\tilde{v}$ in (37), we can compute

$$
\begin{aligned}
Var\left(\tilde{v}|\tilde{c},\tilde{p}\right) &= \frac{1}{\tau_\theta+K\tau_\varepsilon}, \\
E\left(\tilde{v}-\tilde{p}|\tilde{c},\tilde{p}\right) &= \frac{\tau_\theta\bar{\theta}+K\tau_\varepsilon\tilde{S}}{\tau_\theta+K\tau_\varepsilon}+v_c\tilde{c}-\mu\tilde{p}-v_0-\tilde{p}.
\end{aligned}
$$

Inserting the above two expressions into (12) and (33), we obtain the demand functions of A-growers and of B-growers, respectively, as follows:

$$
d_A\left(\tilde{c},\tilde{p}\right) = \frac{\tau_\theta\bar{\theta}+K\tau_\varepsilon\tilde{S}+\left(\tau_\theta+K\tau_\varepsilon\right)\left(v_c\tilde{c}-\mu\tilde{p}-v_0-\tilde{p}\right)}{\alpha}-\left(\tilde{p}-\tilde{c}\right), \tag{A6}
$$

$$
d_B\left(\tilde{c},\tilde{p}\right) = \frac{\tau_\theta\bar{\theta}+K\tau_\varepsilon\tilde{S}+\left(\tau_\theta+K\tau_\varepsilon\right)\left(v_c\tilde{c}-\mu\tilde{p}-v_0-\tilde{p}\right)}{\alpha}-\left(b_0-b_c\tilde{c}\right), \tag{A7}
$$

where in (A7) we have replaced x_B with equation (34).

Financial trader k has information set $\{\tilde{s}_k,\tilde{p}\}$. Price $\tilde{p}$ is still equivalent to the signal $\tilde{s}_p$ in (16). Using Bayes' law, we can compute the conditional moments as follows:

$$
\begin{aligned}
E\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right) &= \bar{\theta}+v_c\bar{c}+\beta_{F,p}\left(\tilde{s}_p-\rho\bar{\theta}-\bar{c}\right)+\beta_{F,s}\left(\tilde{s}_k-\bar{\theta}\right)-\mu\tilde{p}-v_0, \\
Var\left(\tilde{v}|\tilde{s}_k,\tilde{p}\right) &= \frac{\left(K-1\right)\left(\tau_c+\tau_\theta v_c^2+K\tau_\varepsilon v_c^2\right)\rho^2-2K\tau_\varepsilon v_c\left(K-1\right)\rho+K^2\tau_\varepsilon}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)},
\end{aligned}
$$

where

$$
\begin{aligned}
\beta_{F,p} &= K\tau_\varepsilon\frac{\tau_c\left(K-1\right)\rho+Kv_c\left(\tau_\theta+\tau_\varepsilon\right)}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}, \\
\beta_{F,s} &= K\tau_\varepsilon\frac{-v_c\left(\tau_\theta+K\tau_\varepsilon\right)\rho+K\tau_\varepsilon}{\tau_c\left(K-1\right)\left(\tau_\theta+K\tau_\varepsilon\right)\rho^2+K^2\tau_\varepsilon\left(\tau_\theta+\tau_\varepsilon\right)}.
\end{aligned}
$$

Inserting these equations into (15) delivers the expression of $d_F(\tilde{s}_k, \tilde{p})$,

$$d_F(\tilde{s}_k, \tilde{p}) = \frac{\left(1 - \rho\beta_{F,p} - \beta_{F,s}\right)\bar{\theta} + \left(v_c - \beta_{F,p}\right)\bar{c} - v_0 + \rho\beta_{F,p}\tilde{S} + \beta_{F,p}\tilde{c} + \beta_{F,s}\tilde{s}_k - (\mu+1)\tilde{p}}{\gamma Var(\tilde{v}|\tilde{s}_k, \tilde{p})}. \tag{A8}$$

We then plug demand functions (A6), (A7), and (A8) into the market clearing condition (35) and compute

$$\begin{aligned}
\tilde{p} \;=\; & \frac{J\left[\frac{\tau_\theta\bar{\theta} - (\tau_\theta + K\tau_\varepsilon)v_0}{\alpha} - (1-\mu)\,b_0\right] + K\frac{\left(1-\rho\beta_{F,p}-\beta_{F,s}\right)\bar{\theta} + \left(v_c - \beta_{F,p}\right)\bar{c} - v_0}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{(\tau_\theta+K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right) + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}} \\
& + \frac{J\frac{K\tau_\varepsilon}{\alpha} + K\frac{\rho\beta_{F,p}+\beta_{F,s}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{(\tau_\theta+K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right) + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}\tilde{S} \\
& + \frac{J\left(\frac{(\tau_\theta+K\tau_\varepsilon)v_c}{\alpha} + \left[\mu + (1-\mu)\,b_c\right]\right) + K\frac{\beta_{F,p}}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}{J\left(\frac{(\tau_\theta+K\tau_\varepsilon)(\mu+1)}{\alpha} + \mu\right) + \frac{K(\mu+1)}{\gamma Var(\tilde{v}|\tilde{s}_k,\tilde{p})}}\tilde{c}.
\end{aligned}$$

Comparing with the conjectured price function (11), we have equations (39)–(41) in Proposition 5.

Next we compute the production decision of B-growers to generate equations (42) and (43) in Proposition 5. Inserting the demand function (33) into B-growers' objective function yields the indirect utility after trading as follows:

$$U_{B,1}(\tilde{c}, \tilde{p}) = -\exp\left(-\frac{\left[E(\tilde{v} - \tilde{p}|\tilde{c}, \tilde{p})\right]^2}{2Var(\tilde{v}|\tilde{c}, \tilde{p})} - \alpha\left[\tilde{p}x_B - \left(\tilde{c}x_B + \frac{1}{2}x_B^2\right)\right]\right).$$

The exponent in $U_{B,1}(\tilde{c}, \tilde{p})$ is a quadratic form in $\tilde{p}$, which is normally distributed conditional on $\tilde{c}$. By the moment generating function of a non-central Chi square distribution, we can compute the expected utility of B-growers as follows:

$$U_{B,1}(\tilde{c}) = -\sqrt{\frac{Var(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{c})}{Var(\tilde{v} - \tilde{p}|\tilde{c})}}\exp\left(-\alpha\widetilde{CE}_{B,1}\right) \tag{A9}$$

where

$$\begin{aligned}
\widetilde{CE}_{B,1} \;=\; & -\frac{1}{2}\left[1 + \alpha Var(\tilde{p}|\tilde{c}) - \alpha\frac{\left[Cov(\tilde{p}, E(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{c})\,|\tilde{c})\right]^2}{Var(\tilde{v} - \tilde{p}|\tilde{c})}\right]x_B^2 \\
& + \left[p_0 + p_s\bar{\theta} - \frac{h_0 Cov(\tilde{p}, E(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{c})\,|\tilde{c})}{Var(\tilde{v} - \tilde{p}|\tilde{c})} - \left(-p_c + 1 + \frac{h_c Cov(\tilde{p}, E(\tilde{v} - \tilde{p}|\tilde{p}, \tilde{c})\,|\tilde{c})}{Var(\tilde{v} - \tilde{p}|\tilde{c})}\right)\tilde{c}\right]x_B \\
& + \frac{\left[E(\tilde{v} - \tilde{p}|\tilde{c})\right]^2}{2\alpha Var(\tilde{v} - \tilde{p}|\tilde{c})},
\end{aligned} \tag{A10}$$

with

$$\begin{aligned}
Var\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c}\right) &= Var\left(\tilde{\theta}|\tilde{S}\right)=\frac{1}{\tau_\theta+K\tau_\varepsilon},\\
Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right) &= \frac{1}{\tau_\theta+K\tau_\varepsilon}+\left(\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}-(\mu+1)\,p_s\right)^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right),\\
Var\left(\tilde{p}|\tilde{c}\right) &= p_s^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right),\\
Cov\left(\tilde{p},E\left(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c}\right)|\tilde{c}\right) &= p_s\left(\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon}-(\mu+1)\,p_s\right)\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right),\\
E\left(\tilde{v}-\tilde{p}|\tilde{c}\right) &= h_0+h_c\tilde{c},\\
h_0 &= \left[1-(\mu+1)\,p_s\right]\bar{\theta}-v_0-(\mu+1)\,p_0,\\
h_c &= v_c-(\mu+1)\,p_c.
\end{aligned}$$

Thus, B-growers' optimal production decision is determined by maximizing $\widetilde{CE}_{B,1}$ in (A10). The first-order condition yields

$$x_B=\frac{p_0+p_s\bar{\theta}-\frac{h_0Cov(\tilde{p},E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}{1+\alpha Var\left(\tilde{p}|\tilde{c}\right)-\alpha\frac{[Cov(\tilde{p},E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}-\frac{\left(-p_c+1+\frac{h_cCov(\tilde{p},E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)}{1+\alpha Var\left(\tilde{p}|\tilde{c}\right)-\alpha\frac{[Cov(\tilde{p},E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}}\tilde{c}. \quad \text{(A11)}$$

Comparing (A11) with the conjectured strategy (34) leads to equations (42) and (43) in Proposition 5. Also note that the second-order condition of maximizing $\widetilde{CE}_{B,1}$ is always satisfied by Cauchy–Schwarz inequality.

Expressions of Welfare Variables in the Extended Economy in Section 7

Consumers. Inserting the demand function (2) into consumers' objective function (1) and taking expectation yields

$$CE_{C,0}=\frac{1}{2}E\left[\left(\tilde{\theta}-\tilde{v}\right)^2\right]=\frac{1}{2}\left[Var\left(\tilde{\theta}-\tilde{v}\right)+\left(\bar{\theta}-\bar{v}\right)^2\right],$$

where by equations (11) and (37),

$$\begin{aligned}
\bar{v} &= E\left(\tilde{v}\right)=\bar{\theta}+v_c\bar{c}-\mu\bar{p}-v_0,\\
\bar{p} &= E\left(\tilde{p}\right)=p_0+p_s\bar{\theta}+p_c\bar{c},\\
Var\left(\tilde{\theta}-\tilde{v}\right) &= Var\left(\mu\tilde{p}-v_c\tilde{c}\right)=\mu^2Var\left(\tilde{p}\right)+\left(v_c-2\mu p_c\right)\frac{v_c}{\tau_c},\\
Var\left(\tilde{p}\right) &= p_s^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right)+p_c^2\frac{1}{\tau_c}.
\end{aligned}$$

Financial traders. Their certainty equivalent is still given by equation (29), i.e.,

$$CE_{F,0} = \frac{[E(\tilde{v}-\tilde{p})]^2}{2\gamma Var(\tilde{v}-\tilde{p})} + \frac{1}{2\gamma}\log\left[\frac{Var(\tilde{v}-\tilde{p})}{Var(\tilde{v}|\tilde{s}_k,\tilde{p})}\right],$$

where

$$\begin{aligned}
E(\tilde{v}-\tilde{p}) &= \bar{v}-\bar{p}, \\
Var(\tilde{v}-\tilde{p}) &= \left(\frac{1}{\tau_\theta}+\frac{v_c^2}{\tau_c}\right) + (\mu+1)^2 Var(\tilde{p}) - 2(\mu+1)\left(\frac{p_s}{\tau_\theta}+\frac{v_c p_c}{\tau_c}\right), \\
Var(\tilde{v}|\tilde{s}_k,\tilde{p}) &= \frac{(K-1)(\tau_c+\tau_\theta v_c^2+K\tau_\varepsilon v_c^2)\rho^2 - 2K\tau_\varepsilon v_c(K-1)\rho + K^2\tau_\varepsilon}{\tau_c(K-1)(\tau_\theta+K\tau_\varepsilon)\rho^2 + K^2\tau_\varepsilon(\tau_\theta+\tau_\varepsilon)}.
\end{aligned}$$

***A*-growers.** Their indirect utility after trading and production is still given by equation (30), i.e.,We still have the Date-1 indirect utility:

$$\begin{aligned}
U_A(\tilde{c},\tilde{p}) &= -\exp\left\{-\alpha\left[\frac{[E(\tilde{v}-\tilde{p}|\tilde{c},\tilde{p})]^2}{2\alpha Var(\tilde{v}|\tilde{c},\tilde{p})} + \frac{1}{2}(\tilde{p}-\tilde{c})^2\right]\right\} \\
&= -\exp\left\{-\frac{[E(\tilde{v}-\tilde{p}|\tilde{c},\tilde{p})]^2}{2Var(\tilde{v}|\tilde{c},\tilde{p})} - \frac{\alpha}{2}(\tilde{p}-\tilde{c})^2\right\}.
\end{aligned}$$

Both $E(\tilde{v}-\tilde{p}|\tilde{c},\tilde{p})$ and $(\tilde{p}-\tilde{c})$ are normally distributed. So, applying the moment generating function of a non-central Chi squared distribution, we can compute the ex-ante certainty equivalent of A-growers

$$\begin{aligned}
CE_{A,0} &= -\frac{1}{\alpha}\log\left(-E\left[U_A(\tilde{c},\tilde{p})\right]\right) \\
&= \frac{1}{\alpha}\left(Q_0 - \frac{1}{2}Q_1'(I+2\Sigma Q_2)^{-1}\Sigma Q_1\right) + \frac{1}{2\alpha}\log|I+2\Sigma Q_2|,
\end{aligned}$$

where

$$\begin{aligned}
Q_2 &= \begin{bmatrix} \frac{\tau_\theta+K\tau_\varepsilon}{2} & 0 \\ 0 & \frac{\alpha}{2} \end{bmatrix}, Q_1 = \begin{bmatrix} (\tau_\theta+K\tau_\varepsilon)(\bar{v}-\bar{p}) \\ \alpha(\bar{p}-\bar{c}) \end{bmatrix}, \\
Q_0 &= \frac{(\tau_\theta+K\tau_\varepsilon)(\bar{v}-\bar{p})^2}{2} + \frac{\alpha}{2}(\bar{p}-\bar{c})^2,
\end{aligned}$$

and Σ is covariance matrix of $(E(\tilde{v}-\tilde{p}|\tilde{c},\tilde{p}), \tilde{p}-\tilde{c})'$, with elements given by

$$\begin{aligned}
\Sigma_{11} &= \left[\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon} - (\mu+1)p_s\right]^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right) + [v_c-(\mu+1)p_c]^2\frac{1}{\tau_c}, \\
\Sigma_{12} &= \Sigma_{21} = \left[\frac{K\tau_\varepsilon}{\tau_\theta+K\tau_\varepsilon} - (\mu+1)p_s\right]p_s\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right) + [v_c-(\mu+1)p_c](p_c-1)\frac{1}{\tau_c}, \\
\Sigma_{22} &= p_s^2\left(\frac{1}{\tau_\theta}+\frac{1}{K\tau_\varepsilon}\right) + (p_c-1)^2\frac{1}{\tau_c}.
\end{aligned}$$

B**-growers.** Inserting the optimal production (A11) into the expression of $\widetilde{CE}_{B,1}$ in (A10), we have

$$\widetilde{CE}^*_{B,1} = \frac{\left[p_0 + p_s\bar{\theta} - \frac{h_0 Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})} - \left(-p_c + 1 + \frac{h_c Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)\tilde{c}\right]^2}{2\left[1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right]} + \frac{\left[E\left(\tilde{v}-\tilde{p}|\tilde{c}\right)\right]^2}{2\alpha Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right)},$$

which is quadratic in $\tilde{c}$. By (A9), the ex-ante certainty equivalent of B-growers is given by

$$CE_{B,0} = \frac{1}{2\alpha}\log\left(\frac{Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right)}{Var\left(\tilde{\theta}|\tilde{S}\right)}\right) - \frac{1}{\alpha}\log\left(E\left[\exp\left(-\alpha\widetilde{CE}^*_{B,1}\right)\right]\right).$$

Direct computation shows

$$E\left[\exp\left(-\alpha\widetilde{CE}^*_{B,1}\right)\right] = \frac{1}{\sqrt{1+2A_2/\tau_c}}\exp\left\{-\frac{2A_2\bar{c}^2 - 2A_1\bar{c} - A_1^2/\tau_c}{2\left(1+2A_2/\tau_c\right)} + A_0\right\},$$

where

$$A_2 = \alpha\left(\frac{\left(-p_c + 1 + \frac{h_c Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)^2}{2\left(1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)} + \frac{h_c^2}{2\alpha Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right)}\right),$$

$$A_1 = -\alpha\left(\frac{-\left(p_0 + p_s\bar{\theta} - \frac{h_0 Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)\left(-p_c + 1 + \frac{h_c Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)}{1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}} + \frac{h_0 h_c}{\alpha Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right)}\right),$$

$$A_0 = -\alpha\left(\frac{\left(p_0 + p_s\bar{\theta} - \frac{h_0 Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)^2}{2\left(1 + \alpha Var\left(\tilde{p}|\tilde{c}\right) - \alpha\frac{[Cov(\tilde{p}, E(\tilde{v}-\tilde{p}|\tilde{p},\tilde{c})|\tilde{c})]^2}{Var(\tilde{v}-\tilde{p}|\tilde{c})}\right)} + \frac{h_0^2}{2\alpha Var\left(\tilde{v}-\tilde{p}|\tilde{c}\right)}\right),$$

where the expressions of the conditional moments and h_0 and h_c are given in Proposition 5.

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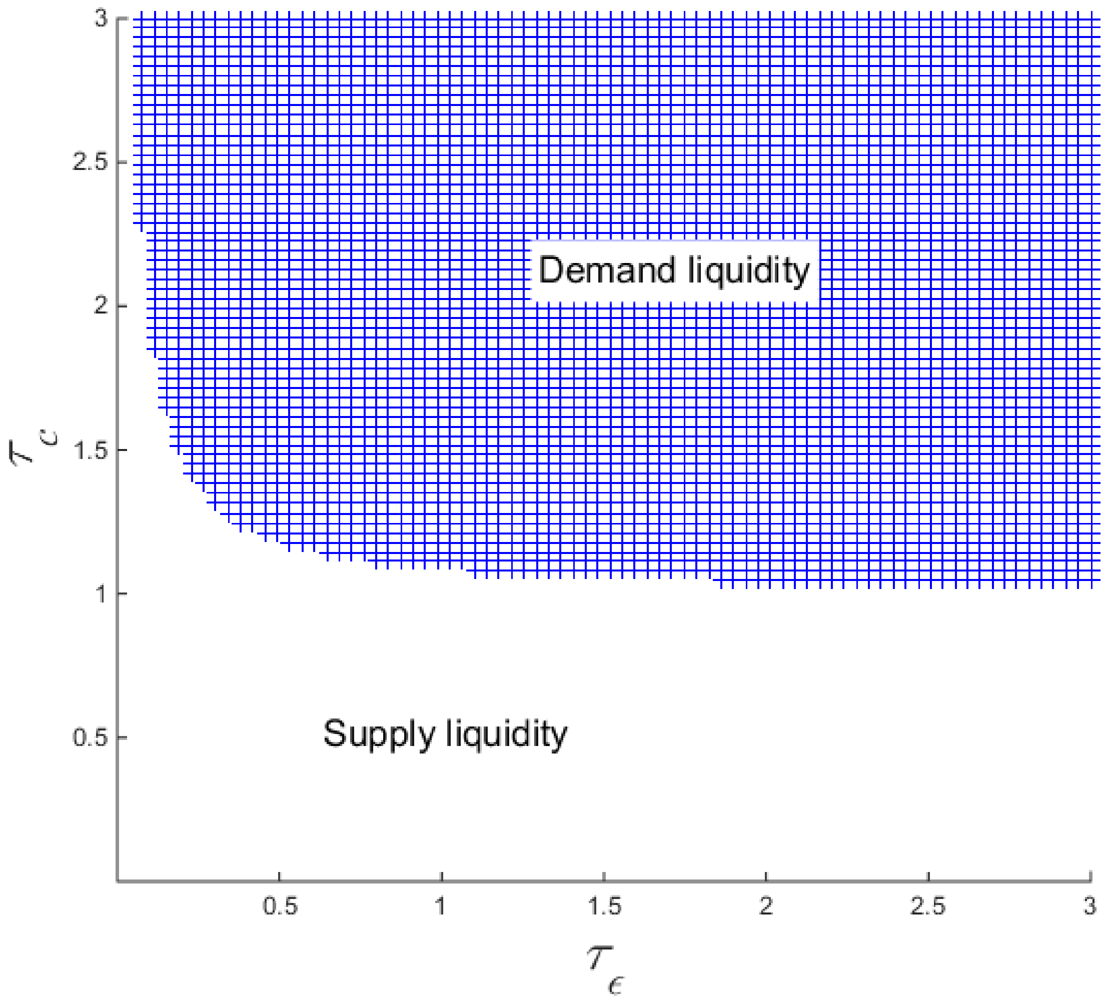
Figure 1: Timeline

- Financial trader k observes private information $\tilde{s}_k$
- Growers observe private information $\tilde{c}$
- Financial markets open, and financial traders and growers trade futures contracts at price $\tilde{p}$
- Growers make production decisions

- Consumers observe the preference shock $\tilde{\theta}$
- Spot market opens, and consumers and growers trade wheat at price $\tilde{v}$
- Cash flows are realized and all agents consume

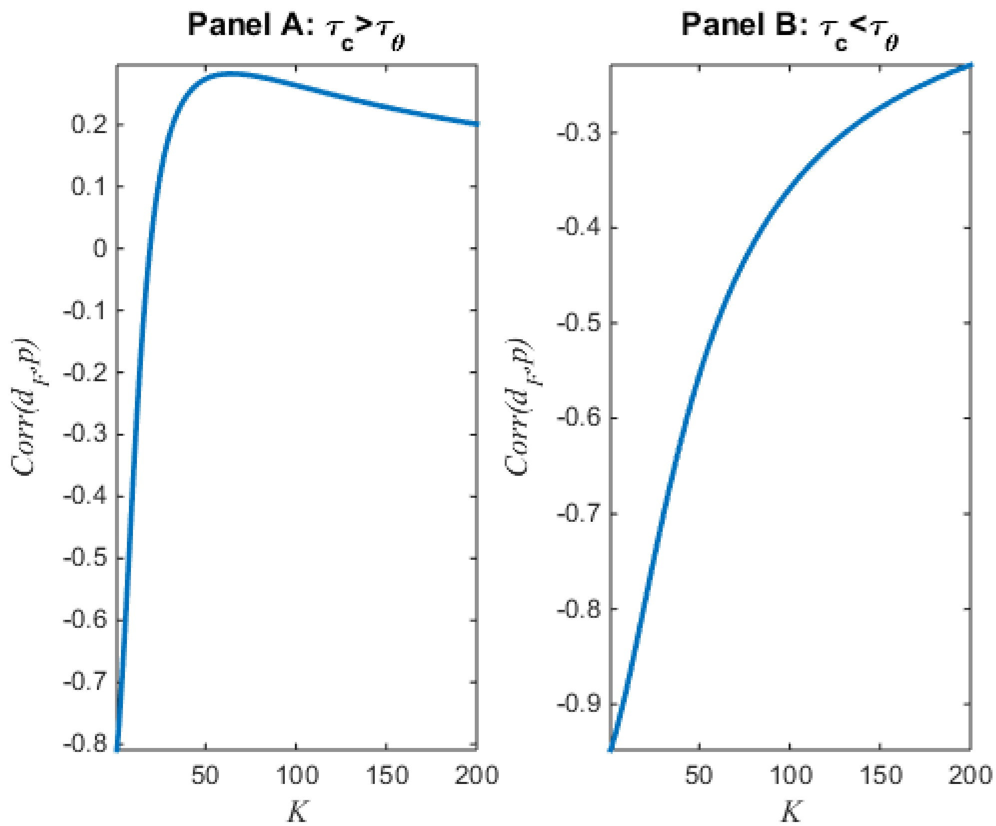
Note: This figure plots the order of events in the economy.

Figure 2: The Region of Financial Traders as Liquidity Suppliers/Demanders



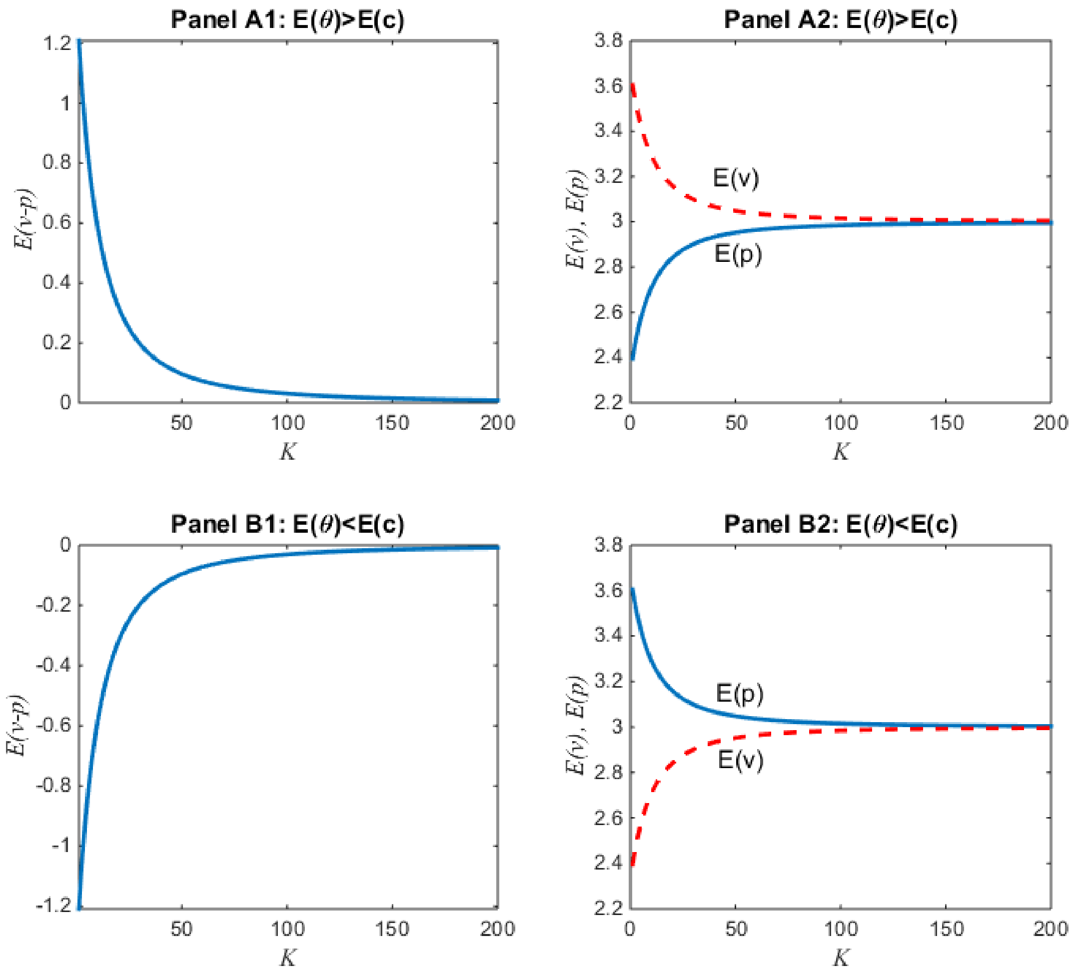
Note: The symbol "+" indicates the region for which financial traders demand liquidity in equilibrium. Financial traders demand liquidity if $Cov(d_F, \tilde{p}) > 0$, and they supply liquidity if $Cov(d_F, \tilde{p}) < 0$. The parameter values are: $\tau_\theta = \alpha = \gamma = 1, J = K = 20, \bar{\theta} = 5$ and $\bar{c} = 1$.

Figure 3: Trading Implications of Commodity Financialization



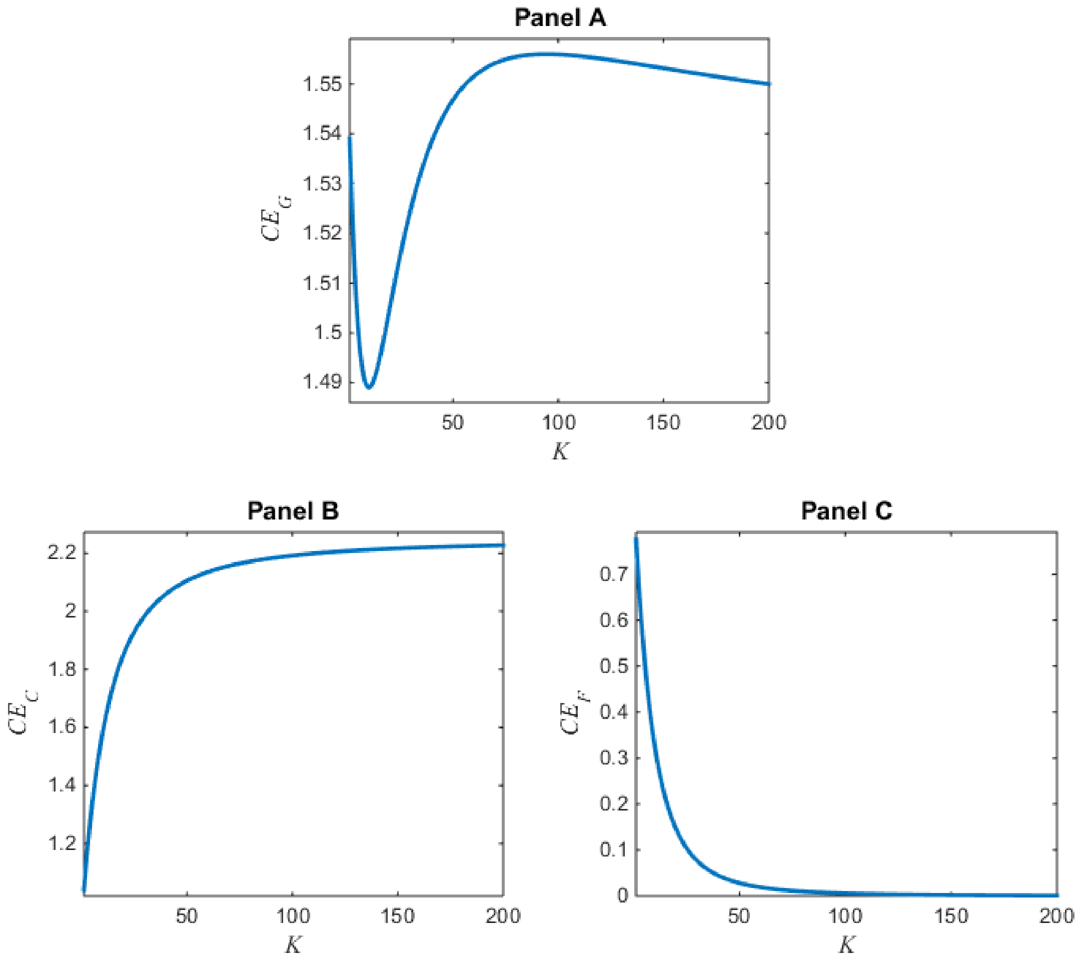
Note: This figure plots $Corr(d_F, \tilde{p})$ against the number K of financial traders. In Panel A, $\tau_c = 2$ and $\tau_\theta = 1$. In Panel B, $\tau_c = 0.5$ and $\tau_\theta = 1$. In both panels, the other parameters are: $\tau_\varepsilon = 0.1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$.

Figure 4: Pricing Implications of Commodity Financialization



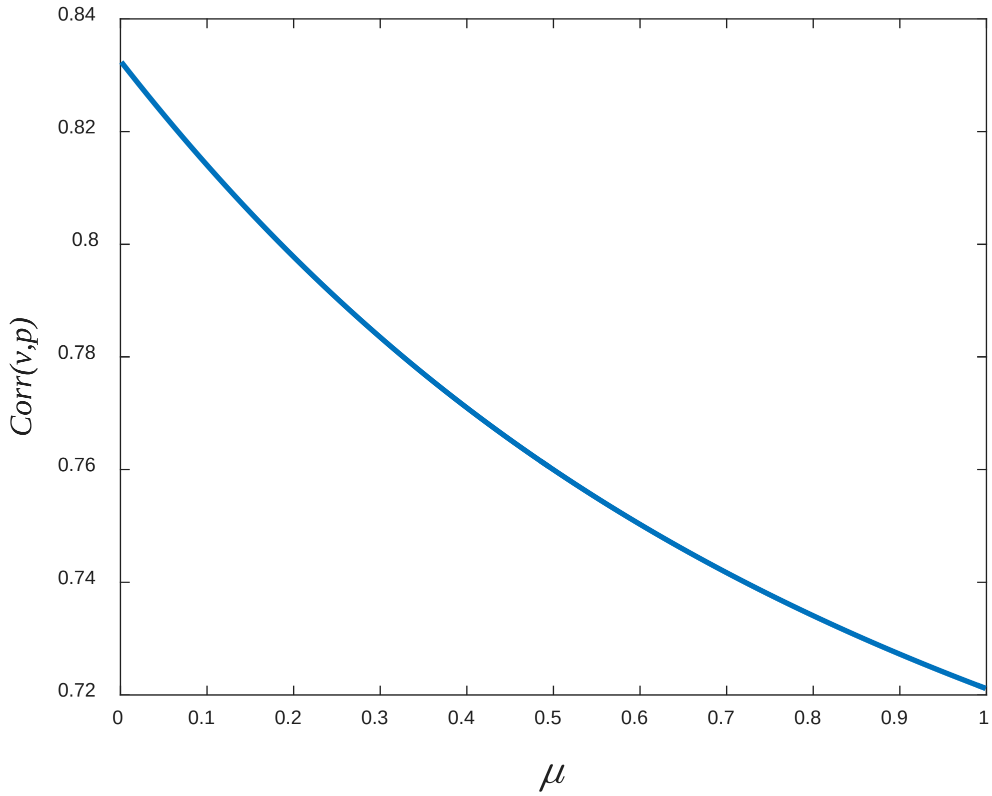
Note: This figure plots the futures price bias $E(\tilde{v}-\tilde{p})$, the average spot price $E(\tilde{v})$, and the average futures price $E(\tilde{p})$ against the number K of financial traders. In Panels A1 and A2, $\bar{\theta}=5$ and $\bar{c}=1$. In Panels B1 and B2, $\bar{\theta}=1$ and $\bar{c}=5$. In all four panels, the other parameters are: $\tau_\varepsilon=0.1$, $\tau_c=\tau_\theta=1$, $\alpha=\gamma=1$ and $J=20$.

Figure 5: Welfare Implications of Commodity Financialization



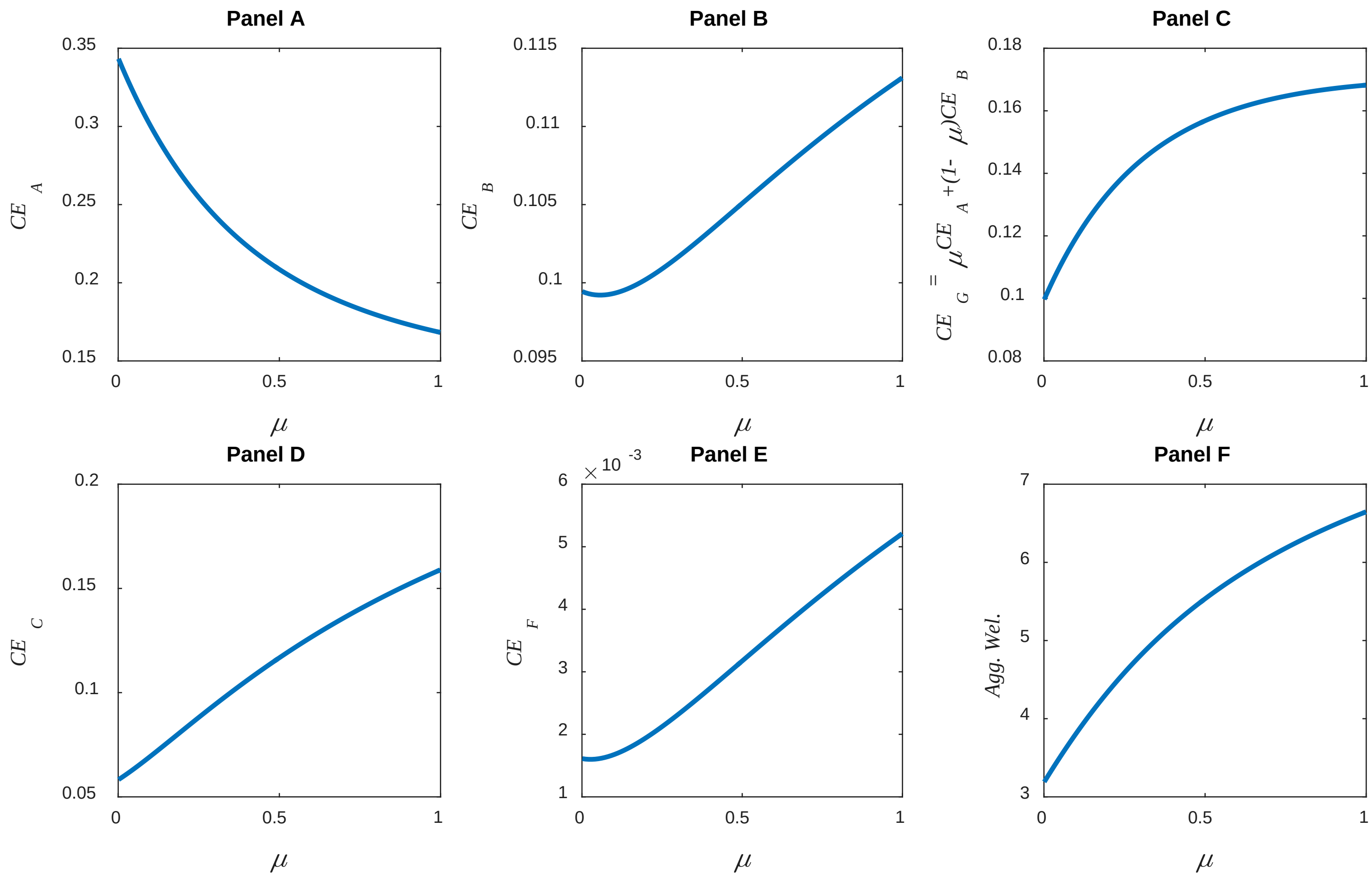
This figure plots how the number K of financial traders affects the welfare of growers (Panel A), the welfare of consumers (Panel B), and the welfare of financial traders (Panel C). The other parameters are: $\tau_\varepsilon = 0.1$, $\tau_c = \tau_\theta = 1$, $\alpha = \gamma = 1$, $J = 20$, $\bar{\theta} = 5$ and $\bar{c} = 1$.

Figure 6: Correlation between Futures and Spot Prices in Extended Economies



This figure plots the correlation coefficient $Corr(\tilde{p}, \tilde{v})$ between the futures price and the spot price in the extended economy with two types of growers. The first type (A-growers) can adjust production after seeing the futures price. The fraction of the first type of growers is μ which measures the strength of the feedback effect from the futures price to the later spot price. The second type (B-growers) of growers has to make production decisions before the futures market is open. The other parameters are: $\tau_{\varepsilon} = 0.1$, $\tau_c = \tau_{\theta} = \alpha = \gamma = \bar{\theta} = \bar{c} = 1$ and $J = K = 20$.

Figure 7: Welfare of Agents in Extended Economies



This figure plots various welfare variables in the extended economy with two types of growers. The first type (A-growers) can adjust production after seeing the futures price. The fraction of the first type of growers is μ, which measures the strength of the feedback effect. The second type of growers (B-growers) has to make production decisions before the futures market is open. Variables CE_A, CE_B, CE_C, and CE_F are the certainty equivalent of A-growers, B-growers, consumers, and financial traders. Aggregate welfare is the sum of the certainty equivalents of all agents. The other parameters are: $\tau_\varepsilon = 0.1, \tau_c = \tau_\theta = \alpha = \gamma = \bar{\theta} = \bar{c} = 1$ and $J = K = 20$.